TOWARDS BAYESIAN ESTIMATOR SELECTION FOR QUIKSCAT WIND AND RAIN ESTIMATION

Michael P. Owen and David G. Long

Brigham Young University, MERS Laboratory, 459 CB, Provo UT 84602

ABSTRACT

The QuikSCAT scatterometer infers wind vectors over the ocean using measurements of the surface backscatter. During rain events the QuikSCAT observations are subject to rain contamination. Three separate estimators have been developed: wind-only, simultaneous wind and rain, and rain-only, which account for rain contamination in varying degrees. This paper introduces a Bayes estimator selection technique to adaptively choose a best estimator from among the three types of estimators at each measurement location. Bayes estimator selection is introduced from a general perspective after which it is applied specifically to QuikSCAT wind and rain estimation. Bayes estimator selection is demonstrated in a case study to illustrate improvements in wind and rain estimation which can be obtained.

***Index Terms*—** QuikSCAT, rain, wind, decision theory, estimation

1. INTRODUCTION

Wind and rain estimation over the ocean is possible using data provided by the QuikSCAT scatterometer. The QuikSCAT scatterometer measures the radar cross section or backscatter of the ocean and uses a model function to infer the most likely wind vector to have produced the observed measurements [1]. During rainy conditions, estimated to affect 4 to 10% of all QuikSCAT observations, backscatter measurements are contaminated due to additional scattering from rain and the resulting wind estimates are contaminated. To account for these rain effects three slightly different estimation techniques have been developed: wind-only [2], simultaneous-wind-rain [3, 4, 5, 6], and rain-only estimation [7]. The performance of each estimator is dependent on the underlying wind-rain conditions. As such, each estimation technique is best under certain backscatter conditions and no single technique is suitable for all conditions. However, by adaptively selecting the estimator most appropriate to the true conditions, performance can surpass that of any individual estimator. In this paper we introduce Bayes estimator selection, a technique whereby a single best estimator is selected for each case, and then apply the technique to QuikSCAT wind and rain estimation.

2. M-ARY ESTIMATOR SELECTION

M-ary Bayes estimator selection is a modification of Bayes decision theory which attempts to select a single 'best' estimate from M viable candidate estimates. In the ideal case, the 'best' estimate is the estimate which has minimum squared error to the true vector. For most interesting estimation situations the true vector is unknown and so some approximation must be used. Bayes estimator selection is based upon the idea that selecting the estimate with minimum *expected* squared-error is a good approximation to the ideal minimum squared-error estimate.

The Bayes decision theory mechanism [8] is an ideal framework with which to construct such an estimator selection rule. The Bayes risk function for a true parameter distribution F_θ and a decision rule ϕ_j can be written as

$$r(F_\theta, \phi_j) = \int_\theta \sum_{i=1}^{M} L[\vartheta, \phi_j(\mathbf{x}_i)] f_{\mathbf{X}|\theta}(\mathbf{x}_i|\vartheta) f_\theta(\vartheta) d\theta \qquad (1)$$

where θ is a random variable representing the true conditions which has realizations ϑ and a pdf $f_\theta(\vartheta)$. $\mathbf{X}_i$ represents the observation random variable with realizations indicated by $\mathbf{x}_i$. $L[\vartheta, \phi_j(\mathbf{x}_i)]$ is a loss function representing the cost of the jth decision rule ϕ_j based on the observation $\mathbf{x}_i$ and true condition ϑ. $f_{\mathbf{X}_i|\theta}(\mathbf{x}_i|\vartheta)$ is the conditional pdf of the observation random variable $\mathbf{X}$ conditioned on the true parameter θ. M refers to the number of realizations of the observation random variable which contribute to a single decision.

This framework can be adapted for estimator selection by appropriately defining each component in Eq. 1. For estimator selection the largest adjustment is that the observations $\mathbf{x}_i$ are estimates of the realization of the parameter ϑ. Thus we exchange the notation of $\mathbf{x}_i$ for $\hat{\vartheta}_i$. This change is a fundamental difference from traditional Bayes decision theory and cannot be made lightly. In traditional Bayes decision theory the typical approach is to make a decision about what the realization of the underlying parameter θ is, where ϑ is typically drawn from a discrete space. In this new application, the purpose is to estimate the realization ϑ of the parameter θ which is a member of a continuous space. In essence, Bayes estimator selection fundamentally changes the meaning of the decision mechanism.

With this in mind, the decision rule $\phi_j(\vartheta_i)$ can be interpreted as the decision to accept the estimate ϑ_j as the 'best' estimate based on the observation of the estimate ϑ_i. The interpretation of the conditional pdf must also be defined. For estimator selection we can interpret the conditional pdf $f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta))$ to represent the probability that the estimate $\hat{\vartheta}_j$ has minimum squared-error given the true conditions.

Recall that the objective is to select a 'best' estimate, i.e. the estimate which has minimum expected-squared-error. This can be achieved by defining an appropriate loss function

$$L[\vartheta, \phi_j(\hat{\vartheta}_i)] = (\vartheta - \hat{\vartheta}_i)^T N(\vartheta - \hat{\vartheta}_i)(\kappa_j \delta_{ij} + \tau_j(1 - \delta_{ij})) \quad (2)$$

Here N is a diagonal normalization matrix to appropriately weight the components of the vector ϑ. κ_j and τ_j are also weighting terms to weight the loss function based on which estimate and which decision rule are used. κ_j determines the weight of the loss when the estimate is that selected by the decision rule ϕ_j and τ_j gives the weight of the loss when the estimate is not selected by the decision rule.

With the appropriately defined loss function the Bayes risk can be written after some simplification as

$$\begin{aligned} r(F_\theta, \phi_j) &= \int_\theta (\vartheta - \hat{\vartheta}_j)^T N(\vartheta - \hat{\vartheta}_j) \\ &\quad \Big(\tau_j(1 - f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta)) + \kappa_j f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta)\Big) \\ &\quad f_\theta(\vartheta) d\vartheta. \end{aligned} \quad (3)$$

Defining the probability that the estimate $\hat{\vartheta}_j$ is not best given the parameter realization ϑ as

$$f_{\hat{\vartheta}|\theta}(\sim \hat{\vartheta}_j|\vartheta) = 1 - f_{\hat{\vartheta}|\theta}(\hat{\vartheta}_j|\vartheta), \quad (4)$$

the Bayes risk can be written in terms of the expectations over the random variable θ after simplifying as

$$\begin{aligned} r(F_\theta, \phi_j) &= \tau_j E_{\theta|\sim\hat{\vartheta}}[(\vartheta - \hat{\vartheta}_j)^T N(\vartheta - \hat{\vartheta}_j)] f_{\hat{\vartheta}}(\sim \hat{\vartheta}_j) \\ &\quad + \kappa_j E_{\theta|\hat{\vartheta}}[(\vartheta - \hat{\vartheta}_j)^T N(\vartheta - \hat{\vartheta}_j)] f_{\hat{\vartheta}}(\hat{\vartheta}_j). \end{aligned} \quad (5)$$

In this notation the Bayes risk is a linear combination of the conditional posterior expected squared error given that the estimate is best and given that the estimate is not best, where the weights are $\tau_j f_{\hat{\vartheta}}(\sim \hat{\vartheta}_j)$ and $\kappa_j f_{\hat{\vartheta}}(\hat{\vartheta}_j)$. The dependence of the weights on the estimate type, as indexed by j, can be removed by redefining τ_j and κ_j with a normalization factor as

$$\tau_j = \frac{\tau}{f_{\hat{\vartheta}}(\sim \hat{\vartheta}_j)} \quad (6)$$

$$\kappa_j = \frac{\kappa}{f_{\hat{\theta}}(\hat{\vartheta}_j)} \quad (7)$$

with the additional constraint that $\tau + \kappa = 1$.

The normalized weighting coefficients reduce the Bayes risk for estimator selection to

$$\begin{aligned} r(F_\theta, \phi_j) &= \tau E_{\theta|\sim\hat{\vartheta}}[(\vartheta - \hat{\vartheta}_j)^T N(\vartheta - \hat{\vartheta}_j)] \\ &\quad + \kappa E_{\theta|\hat{\vartheta}}[(\vartheta - \hat{\vartheta}_j)^T N(\vartheta - \hat{\vartheta}_j)]. \end{aligned} \quad (8)$$

When $\tau = 1$ and $\kappa = 0$ the Bayes risk is the posterior expected squared error for the estimate given that the estimate is not best. This in turn implies that choosing the estimate with minimum Bayes risk when $\tau = 1$ is equivalent to minimizing the estimation error associated with incorrect estimator selections.

On the other hand, when $\kappa = 1$ and $\tau = 0$ the Bayes risk is the posterior expected squared error for the estimate given that the estimate is best. Choosing the estimate with the minimum Bayes risk when $\kappa = 1$ is equivalent to minimizing the estimation error associated with correct estimator selections.

With this interpretation, the choice of τ and κ for Bayes estimator selection can be related to the estimation noise. If estimation noise is high, i.e. the error associated with each estimator is comparable, then minimizing the error associated with incorrect decisions is a good approach to estimator selection. If estimation noise is low, i.e. the best estimate has significantly lower error than the other estimates, then minimizing the cost associated with the correct decision is a good approach to estimator selection. The optimal choice for the weighting coefficients τ and κ is not pursued in this paper.

After choosing the weights τ and κ, selecting the estimate with minimum expected-squared-error is by definition selecting the estimate which minimizes Eq. 8. Thus selecting one of M candidate estimates as the 'best' estimate is reduced to choosing the estimate which has the minimum Bayes risk.

3. APPLICATION TO WIND AND RAIN ESTIMATION

Adopting the proposed Bayes estimator selection mechanism to QuikSCAT wind and rain estimation is relatively straightforward once the parts of Eq. 3 have been appropriately defined in the context of wind and rain estimation. For this case the parameter θ represents the wind and rain vector random variable. Each realization ϑ corresponds to a realization of a true wind and rain vector. The estimates of ϑ, $\hat{\vartheta}_i$, correspond to the wind-only, simultaneous wind-rain, and rain-only estimates indexed by $i = 1, 2, 3$ respectively.

For this short paper we omit a thorough treatment of the prior distributions $f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta))$ and $f_\theta(\vartheta)$ for space considerations. The estimator selection technique is not particularly sensitive to a specific prior modeling technique as the priors generally represent the observed wind and rain distribution and the estimator performance.

The wind and rain prior, $f_\theta(\vartheta)$, can be defined using a histogram of observed wind and rains. The definition of the estimator performance prior, $f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta))$, is somewhat more complicated. For wind and rain estimation there is no closed form for the conditional density of the estimates given the true wind vector. However, the estimator performance prior can be approximated using Monte-Carlo simulations.

We similarly omit a formal derivation of the normalization matrix N and the weighting coefficients τ and κ for space

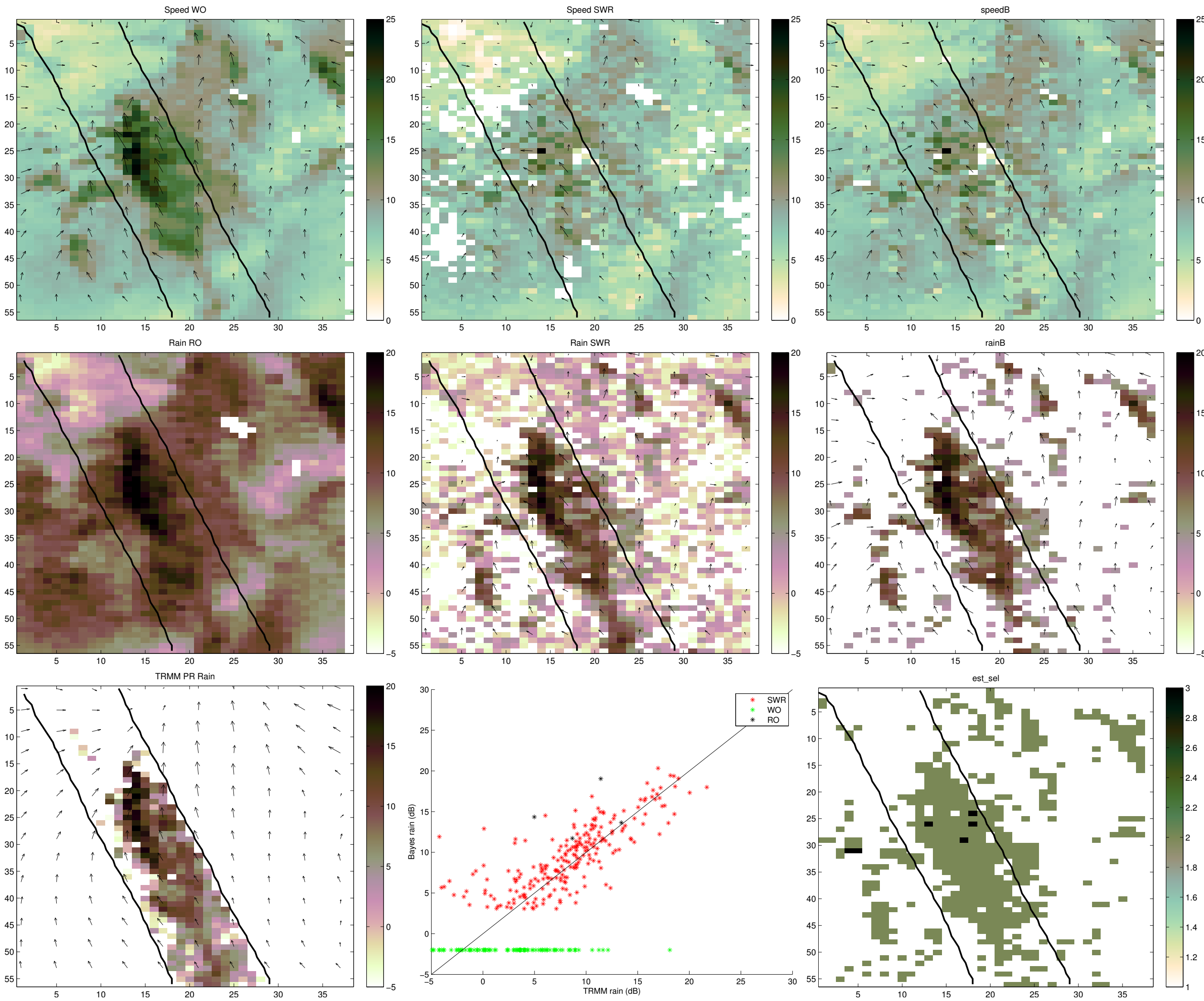

Fig. 1. Estimator selection results for a single case. Upper left: WO estimates. Upper center: SWR wind estimates. Upper right: Bayes selected wind estimates. Middle left: RO estimates. Middle center: SWR rain estimates. Middle right: Bayes selected rain estimates. Lower left: TRMM PR measured rain with NCEP wind vectors overlaid. Lower center: TRMM PR measured rain v. Bayes selected rain estimates. Lower right: Bayes estimator selections. Wind estimates are in m/s and rain estimates in dB km-mm/hr.

considerations in this paper. For reference these values are $\tau = 0.26$, $\kappa = 0.74$, and N is a 3×3 diagonal matrix with $[1/50^2, 0, 1/250^2]$ as the diagonal elements. The weighting coefficients are defined to weight the wind and rain error by the relative importance of each type of error. With the specified weighting coefficients the wind error has a relatively greater contribution to the objective function error than the rain error. The normalization matrix N is defined such that the wind error is weighted by the inverse of the square of the maximum retrievable wind speed (50 m/s), direction error is ignored to simplify ambiguity selection, and rain error is weighted by the inverse of the square of an upper bound on retrievable rain rate (250 km-mm/hr).

4. CASE STUDY

To demonstrate the potential utility for Bayes estimator selection in the context of QuikSCAT wind and rain estimation, we consider the following case study. Figure 1 shows the results of the estimator selection process for a single case study. First note that the rain contamination of the wind-only estimates is visible as high wind estimates corresponding to high TRMM PR measured rain rates. There is a similar effect in the rain-only estimates due to wind contamination in places where there is no rain. The simultaneous wind and rain estimates have more reasonable wind and rain estimates in raining conditions but have no wind estimates in many locations where there is no rain. The Bayes estimator selected wind and rain estimates demonstrate the strengths of each of the estimators while ameliorating their limitations. Thus the Bayes selected wind estimates have little rain contamination and have good performance in non-raining areas. Additionally, the Bayes selected rain estimates correlate well with the measured TRMM PR rain rates and have relatively few missed detections of significant rain events.

The principle advantage of Bayes estimator selection is that it selects the estimator which has minimum error for the true conditions thereby taking advantage of the each type of estimator when it is most appropriate. When the wind-only estimator is best, for non-raining and low rain conditions, Bayes estimator selection consistently selects the wind-only estimate. When the rain is moderate to high and the simultaneous wind and rain estimator is best, Bayes estimator selection chooses the simultaneous wind and rain estimates. When the rain is so strong that the wind signal is not observable, Bayes estimator selection chooses the rain-only estimate.

5. CONCLUSIONS

In summary, Bayes estimator selection has the potential to improve scatterometer wind estimation by selectively choosing wind and rain estimation techniques such that the estimators are used in conditions for which they are most appropriate. Bayes estimator selection can successfully identify and estimate rain contamination and identify areas where wind estimation is not possible due to dominating rain contamination. This leads to improved wind and rain estimation performance on a global scale and simultaneously gives insights into the limitations of and possible improvements for future scatterometers.

6. REFERENCES

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