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COHERENT MULTI-FREQUENCY-BAND RESOLUTION ENHANCEMENT FOR SYNTHETIC APERTURE RADAR

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ABSTRACT

This paper presents a method whereby the range resolution of multi-frequency-band SAR systems can be enhanced. If multiple signals are coherent and cover disjoint frequency bands, they can be combined into a single signal which can be processed using slightly modified SAR processing algorithms, resulting in an image with a range resolution enhanced by the sum of the constituent bandwidths.

***Index Terms*—** synthetic aperture radar, radar signal processing, radar resolution, image enhancement

1. INTRODUCTION

Synthetic Aperture Radar (SAR) has proven useful in many applications, including reconnaissance, surveillance, mapping, change detection, and environmental studies. The use of multiple frequencies with high resolution augments the utility of SAR in each of these applications. Advances in SAR are consistently opening possibilities for more capable SAR systems, with finer resolution and multiple frequencies. Range resolution is a key performance parameter constantly being improved upon to obtain better and more useful SAR images. The resolution is dependent on the bandwidth of the radar chirp according to the equation

$$\Delta R = c/\left(2 \cdot BW\right) \tag{1}$$

where ΔR is the range resolution, c is the speed of light, and BW is the chirp bandwidth.

Obtaining increased bandwidth is difficult, especially at lower frequencies, due to a number of factors:

1. Many bands of the frequency spectrum are protected and SAR operation is prohibited in those bands [1].
2. Radar hardware, including power amplifiers and antennas, is designed to operate at specific frequencies, and very high bandwidth hardware is expensive.
3. Each increase in bandwidth increases the sampling and data storage requirements.

Multi-band operation offers a method to sidestep some of these issues while increasing the bandwidth, thus improving the resolution. This paper considers an approach to coherently combine two (or more) different frequency channels into a single signal, which can be processed with slightly modified processing algorithms. Combining multiple pulses to improve range resolution is not a new idea. Algorithms for stepped-frequency SAR have been in use for many years. This paper extends the work done by Richard Lord et al. [2, 3, 4]. Many of the restrictions previously placed on combining multiple chirps are greatly relaxed in the algorithm proposed in this paper. Notably, the requirements of overlapping bands [4], identical bandwidths, pulse widths, and chirp rates [3], and the requirement of the time shift being an integer number of samples [3] do not apply.

A system utilizing multi-band operation to provide a large bandwidth, even when traditional design limits prohibit such an arrangement, would prove extremely useful. This paper sets forth the theory that makes such a system possible and presents a SAR system, the SlimSAR, that has been designed to be capable of demonstrating this ability.

Section 2 presents the theory of coherent multi-band resolution enhancement, demonstrating each step with simulated SAR data. The SlimSAR is presented in Section 3, and results from SAR data collected with a precursor system are shown in Section 4.

2. THE THEORY OF COHERENT MULTI-BAND RESOLUTION ENHANCEMENT

Range compression in SAR is achevied through matched filtering. A radar chirp is compressed, giving a target response with resolution measured at the 3 dB point. The resolution is determined by the bandwidth of the signal, as in Eq. 1. This section presents a general development for how two (or more) separate, but coherent, signals spanning disjoint frequency bands, can be combined to give a resolution dependent on the sum of the bandwidths, as in Fig. 2.

$$\Delta R_2 = c/\left(2 \cdot (BW_1 + BW_2)\right) \tag{2}$$

Two separate SAR signals are transmitted with center frequencies f_1 and f_2, and possibly different times, τ_1 and τ_2,

$$s_1(\tau) = w_1\left(\tau - \tau_1\right) \cdot \cos\left(2\pi f_1 \tau + \pi K_{r1}\left(\tau - \tau_1\right)^2\right) \tag{3}$$

$$s_2(\tau) = w_2\left(\tau - \tau_2\right) \cdot \cos\left(2\pi f_2 \tau + \pi K_{r2}\left(\tau - \tau_2\right)^2\right) \tag{4}$$

where τ is fast time, K_{rn} is the chirp rate for each pulse, w_n is the window that defines the chirp length, and τ_n is the delay to the transmit of each chirp. These signals are represented at the left of Fig. 1. The goal is to shift the two signals in time and frequency, as on the right of Fig. 1, such that when coherently combined, range compression can be performed using conventional methods, resulting in improved resolution of the target in the image.

These signals return to the radar after reflecting off a target at range R_0,

$$sr_1(\tau) = A_1 \cdot w_1\left(\tau - \tau_1 - \frac{2R_0}{c}\right) \cdot$$
$$\cos\left(2\pi f_1\left(\tau - \frac{2R_0}{c}\right) + \pi K_{r1}\left(\tau - \tau_1 - \frac{2R_0}{c}\right)^2\right) \tag{5}$$

$$sr_2(\tau) = A_2 \cdot w_2\left(\tau - \tau_2 - \frac{2R_0}{c}\right) \cdot$$
$$\cos\left(2\pi f_2\left(\tau - \frac{2R_0}{c}\right) + \pi K_{r2}\left(\tau - \tau_2 - \frac{2R_0}{c}\right)^2\right) \tag{6}$$

where A_n is the signal amplitude, which may vary for different frequency bands.

The signals are quadrature-demodulated by $\cos(2\pi f_n \tau)$ yielding the phases

$$\phi_1(\tau) = \frac{-4\pi f_1 R(\eta)}{c} + \pi K_{r1}\left[\tau - \tau_1 - \frac{2R(\eta)}{c}\right]^2 \quad (7)$$

$$\phi_2(\tau) = \frac{-4\pi f_2 R(\eta)}{c} + \pi K_{r2}\left[\tau - \tau_2 - \frac{2R(\eta)}{c}\right]^2 \quad (8)$$

where $R(\eta)$ is the range to target which changes with slow time η.

We take the range Fourier transform of each signal individually. This is done using the principle of stationary phase (POSP) in performing the Fourier integration, computed by subtracting $2\pi f_\tau \tau$ from Eq. (7) (where f_τ is range frequency),

$$\phi_{0r} = \frac{-4\pi f_1 R(\eta)}{c} + \pi K_{r1}\left[\tau - \tau_1 - \frac{2R(\eta)}{c}\right]^2 - 2\pi f_\tau \tau \quad (9)$$

Take the derivative with respect to τ

$$\frac{d\phi_{0r}}{d\tau} = 2\pi K_{r1}\left[\tau - \tau_1 - \frac{2R(\eta)}{c}\right] - 2\pi f_\tau = 0 \quad (10)$$

and solve for τ

$$\tau = \tau_1 + \frac{f_\tau}{K_r} + \frac{2R(\eta)}{c} \quad (11)$$

Substitute into Eq. (9) and simplify to get the signal after the range Fourier transform of the signal.

$$\begin{aligned}\phi_{1R} &= \frac{-4\pi f_1 R(\eta)}{c} + \frac{\pi f_\tau^2}{K_{r1}} - 2\pi f_\tau\left(\tau_1 + \frac{2R(\eta)}{c} + \frac{f_\tau}{K_{r1}}\right) \\ &= \frac{-4\pi f_1 R(\eta)}{c} - \frac{\pi f_\tau^2}{K_{r1}} + \frac{-4\pi f_\tau R(\eta)}{c} - 2\pi f_\tau \tau_1 \\ &= \frac{-4\pi (f_1 + f_\tau) R(\eta)}{c} - \frac{\pi f_\tau^2}{K_{r1}} - 2\pi f_\tau \tau_1 \quad (12)\end{aligned}$$

$$\phi_{2R} = \frac{-4\pi (f_2 + f_\tau) R(\eta)}{c} - \frac{\pi f_\tau^2}{K_{r2}} - 2\pi f_\tau \tau_2 \quad (13)$$

The time delay difference is eliminated using a time shift, which in the frequency domain is a complex sinusoidal multiply

$$H_{t1} = e^{j2\pi f_\tau (\tau_1 + t_{p1}/2)} \quad (14)$$

$$H_{t2} = e^{j2\pi f_\tau (\tau_2 - t_{p2}/2)} \quad (15)$$

which leaves us with two temporally registered, standard SAR signals. In addition to the time delay, we shift each signal by half the pulse length $t_{pn}/2$ so that the zero frequency points will line up after the frequency shift in the next step. If each signal is sampled at Nyquist, as in our example, we can artificially increase the fast-time sample rate by zero padding in the frequency domain.

The signals are shifted in frequency such that the bandwidths occupy non-overlapping regions of the frequency support band. This is done in the time domain, thus an inverse FFT is computed for each signal. The signals are then shifted in frequency by a complex sinusoidal multiply in the time domain, for our example this shift would be

$$H_{f1} = e^{-j\pi BW_1 \tau} \quad (16)$$

$$H_{f2} = e^{+j\pi BW_2 \tau} \quad (17)$$

The two signals are summed together into a single signal. The difficulty in processing the data now lies in the fact that the Doppler chirp used for azimuth compression is dependent on the effective mixdown frequency of each individual frequency. For our two signals, the combined azimuth chirp is the sum of the individual azimuth chirps dependent on the two effective mixdown frequencies $f_1 + BW_1$ and $f_2 - BW2$. In the Omega-K algorithm the azimuth compression is done in the two-dimensional frequency domain where the two signals have no overlap making it easy to apply the appropriate compression to each band. Alternatively, the time-domain backprojection calculates the expected phase from a given target in order to focus the image. When two signals are summed, the expected phase is calculated from the sum of the signals. These are two options available to us for focusing the image.

Using simulated data of two point targets, separated by 0.75 meters in slant range, the method developed in this section is tested. A comparison between images formed from a single 250 MHz bandwidth signal with a center frequency of 36 GHz, a single 500 MHz bandwidth signal at 36 GHz, and two coherently combined 250 MHz bandwidth signals at 36 GHz and 46 GHz, is shown in Fig. 3. While the two targets are not distinguishable in the single 250 MHz bandwidth image, they are easily visible in both the 500 MHz image and the coherently combined two signal image.

This method easily lends itself to situations where the radar chirp has to skip a frequency band and when hardware constraints make large bandwidth systems impractical. Because each signal stream can be recorded separately, it sidesteps some of the sampling and storage requirements that greatly contribute to the difficultly and expense of large bandwidth systems.

3. THE SLIMSAR SYSTEM

The SlimSAR is a new advancement in high-performance, small, low-cost, SAR. Using a unique design methodology that extends the work from previous successful systems, such as the NuSAR-B system presented in [5], the flexible SlimSAR uses less power and is made smaller, lighter, and more capable by using techniques and technology developed for the MicroASAR in [6]. This compact design is facilitated by the use of a linear-frequency-modulated continuous-wave (LFM-CW) signal, which allows us to achieve a high signal-to-noise ratio while transmitting with less power. A delayed mix-down chirp is used to sidestep the usual swath-width limitations that accompany LFM-CW operation.

The flexible control software allows us to change the radar parameters in flight. The core L-band unit is connected to an upconverter to work at X-band, or any other desired frequency band. The bandwidth is 660 MHz, giving a resolution of 23 cm. The system is designed with a built-in high quality GPS/IMU motion measurement solution (for motion compensation and image geolocation), a small data link, and a gimbal for the X-band antennas. Including all this, the entire system weighs less than 20 lbs and consumes less than 150 Watts, making is very suitable for use on a number of small unmanned aircraft systems.

Like its predecessor, the SlimSAR operates at multiple bandwidths. The ability to operate at L-band and two separate X-bands simultaneously makes it suitable for testing coherent multi-band resolution enhancement. The two X-bands cover the frequencies 8.9546-9.6146 GHz and 9.934-10.594 GHz. Most targets will have similar returns from both X-bands, making resolution enhancement easier. The addition of signal returns at L-band allow us to experiment with combining the returns from widely disparate frequencies, over which the targets in the scene may have quite different characteristics.

4. SAR IMAGES

With SlimSAR data not yet available, multiple bandwidths of disjoint frequencies are synthesized from X-band NuSAR-B data. The 500 MHz signal is decomposed into 125 MHz chunks and the outer two sub-bands (with a 250 MHz gap) are treated as separate signals. The two bandwidths are shifted and combined, as described in Section 2, to form a single image with a bandwidth of 250 MHz. The results showing improved range resolution in the coherently combined SAR image are seen in Figs. 4 and 5.

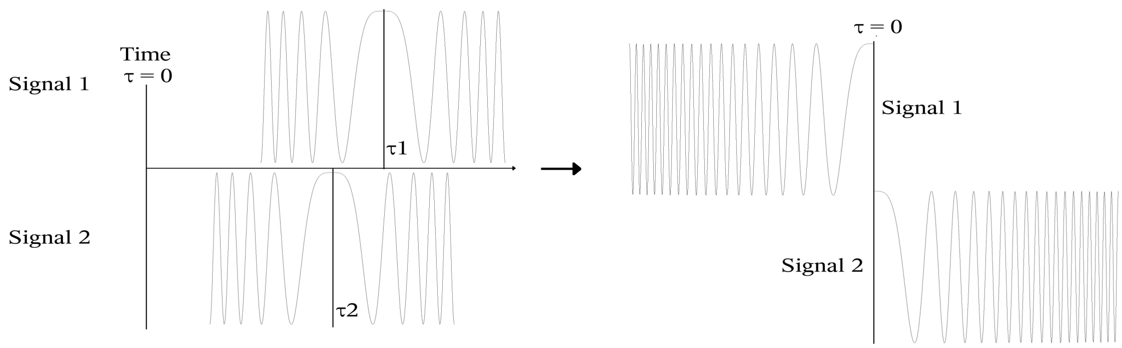


Fig. 1. Two coherent SAR pulses at different center frequencies (shown without the carrier) are transmitted at times τ_1 and τ_2 (left). In order to coherently combine the two signals to increase the total bandwidth, the signals must be shifted in time and frequency such that the timing difference is removed, the zero frequency points of the signals line up, and the signals span separate mixed-down frequency bands (right).

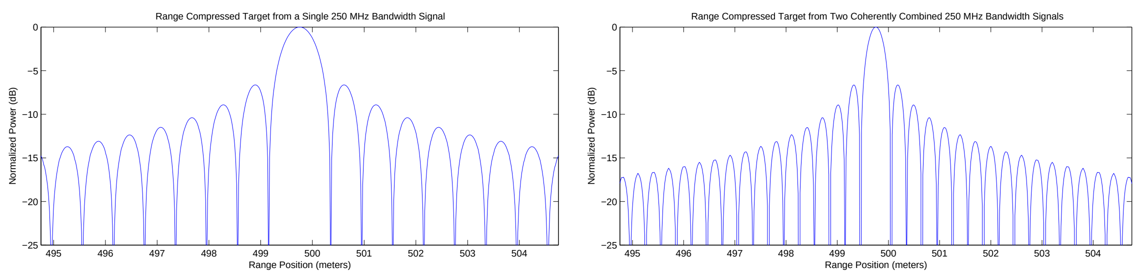


Fig. 2. A simulated point target is range-compressed with a single 250 MHz bandwidth signal (left) and two coherently combined 250 MHz bandwidth signals (right). The measured resolution of the single band image on the left is 57.15 cm while that of the double band image on the right is 28.53 cm.

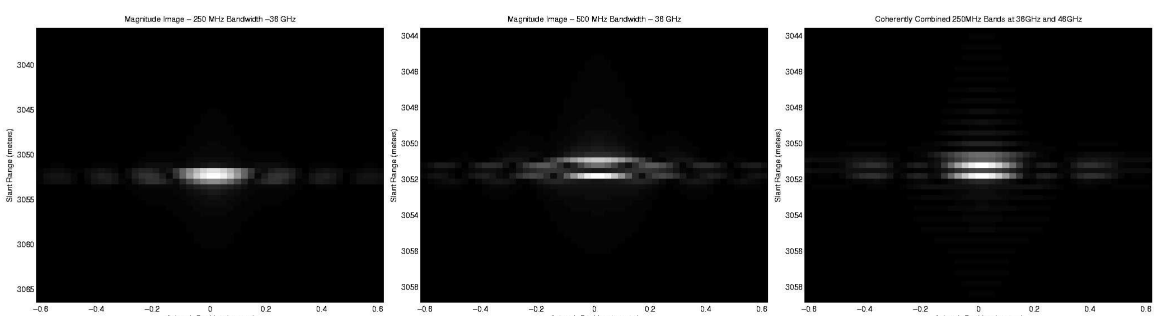


Fig. 3. Fully compressed simulated SAR images of two point targets separated by 0.75 m in slant range. The leftmost image is of single 250 MHz bandwidth data with a center frequency of 36 GHz. The resolution is 60 cm which is not sufficient to clearly separate the two targets. The center image is from single bandwidth data with a resolution of 30 cm at 36 GHz. The image at right is formed from two separate 250 MHz bandwidths, at 36 GHz and 46 GHz, which combine to give an effective bandwidth of 500 MHz. The two targets are clearly visible in the rightmost two images, showing that coherent multi-band resolution enhancement performs similarly to single band data of the same bandwidth. The data is oversampled in the azimuth direction, visually stretching the targets. The azimuth resolution is measured at 16 cm.

5. CONCLUSION

This paper shows that, given certain constraints, multi-frequency-band SAR signals can be combined to improve range resolution. The signals must be coherent, though the timing between signals can be any reasonable known value where the target is temporally coherent. The resolution improvement is achieved by combining the bandwiths to form a larger single bandwidth, thus, for maximum enhancement, the frequency bands must be disjoint. It is also assumed that the target is coherent over the multiple frequency bands. This method can be used to increase the bandwidth given hardware constraints, as an alternative to common notched chirp approaches to keep-out bands, and to explore the phenomenology of combining returns at disparate frequencies.

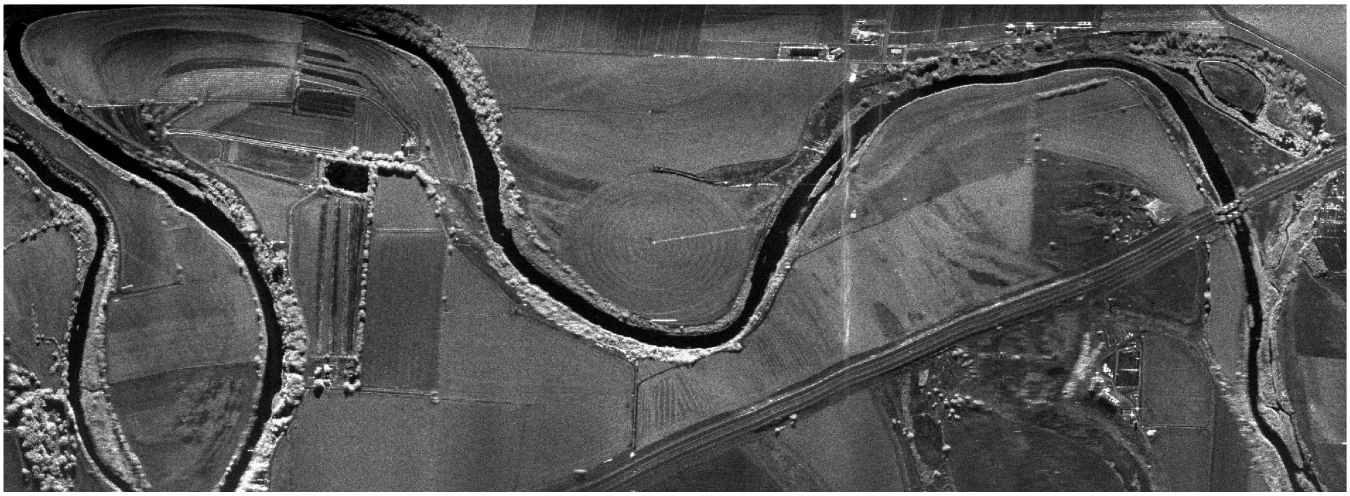
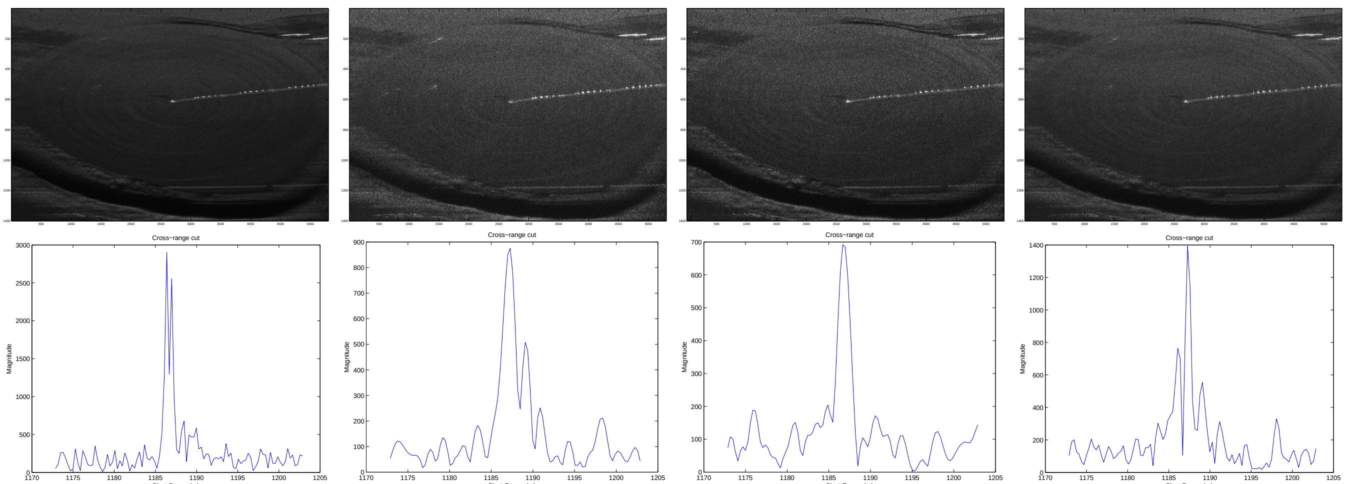


Fig. 4. A 500 MHz bandwidth NuSAR-B image of an agricultural area near Brigham City, Utah. This SAR data is separated into two data sets of 125 MHz bandwidth each, disjoint in frequency with a 250 MHz gap. These two bands are used to demonstrate the techniques set forth in this paper. Figure 5 focuses in on the irrigation wheel line at the center of this image.



(e) Full 500 MHz (9.5-10 GHz) (f) 125 MHz (9.5-9.625 GHz) (g) 125 MHz (9.875-10 GHz) (h) Combined 250 MHz

Fig. 5. A circular field with a rotating irrigation wheel line is shown in the top row of images. The bottom row shows a cross-range cut of the third bright point from the center of the wheel line. The first column shows the results from using the full 500 MHz signal, where (e) shows that there are actually two closely spaced point targets. In (f) and (g) we see that the range resolution is degraded such that the two targets appear as one. The signals from (f) and (g) are combined using the techniques in this paper to form the images in column (h), where we see an improvement in range resolution and we begin to differentiate between the two targets.

6. REFERENCES

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