Development of a Statistical Method for Eliminating Improbable Wind Aliases in Scatterometer Wind Retrieval

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Abstract -- Wind velocities over the ocean can be estimated using measurements from spaceborne scatterometers by inverting the Geophysical Model Function (GMF) which relates normalized backscatter to wind velocity. Current estimation procedures employ maximum-likelihood techniques. Unfortunately, there are several local maxima of the maximum-likelihood function. As a result, several (2-6) wind estimates are returned as possible solutions at each wind vector cell. An ambiguity-removal step is required to determine a wind field. In this paper, we develop a statistical test to distinguish among the maxima of a maximum likelihood equation, and apply it to wind estimation. An upper bound is derived on the probability of error if a lower likelihood wind estimate is discarded. This bound is used to eliminate improbable wind solutions. Using this procedure we show that for most ERS-1 wind vector cells the number of wind estimates can be reduced to two. This reduces the complexity of the ambiguity-removal step while at the same time increasing the confidence in the entire retrieved wind field.

INTRODUCTION

Spaceborne scatterometers are a proven method of estimating wind velocities over the earth's oceans. A wind scatterometer makes indirect measurements of the normalized radar backscatter coefficient, σ^o, of the ocean's surface. The backscatter is then related to the wind velocity over the surface by inverting an empirical relationship called the Geophysical Model Function (GMF). Wind retrieval algorithms are based on the optimization of a maximum-likelihood (ML) objective function which is based on a statistical model of the σ° measurements.

Unfortunately, in general there are multiple wind velocities, or ambiguities, which give maxima of the likelihood function. Current techniques keep all of the local maxima of the likelihood function for further processing by an ambiguity-removal algorithm which uses correlation in adjacent cells or other techniques to determine a wind field.

However, keeping all the wind vectors which give local maxima of the likelihood function does not adhere to the general philosophy of a maximum likelihood estimate: pick the wind velocity which gives the largest probability that the measurements would have been observed. It is hard to statistically justify retaining a wind velocity estimate that has a distinctly lower likelihood value than the maximum value. The problem is determining how much lower the likelihood value should be before the wind velocity solution is eliminated as a possible wind vector. A solution to this problem is to formulate a probabilistic question, and then place a probability threshold for eliminating wind vectors at a statistically satisfying level.

It is important to note that the technique developed below for resolving maxima in a ML equation with multiple maxima can be employed in any ML estimation procedure. The notation given in developing the technique is convenient for wind estimation but the technique is general.

STATISTICAL TEST

To establish notation, let $\mathbf{z}$ be a vector of σ° measurements, and let $\mathbf{w}$ be the wind vector. Let $\mathbf{z}_0$ be a particular vector of measurements and $\mathbf{w}_0$ and $\mathbf{w}_1$ particular wind vectors associated with the maxima of the log-likelihood function, i.e., $\mathbf{w}_0$ and $\mathbf{w}_1$ are two ambiguities, where $\mathbf{w}_1$ is the most-likely ambiguity of the set of ambiguities resulting from point-wise wind retrieval. In addition, let $M_{k,i}$ be the value of the Geophysical Model Function (GMF) when the wind is $\mathbf{w}_i$ and the measurement is taken with the incidence and azimuth angles corresponding to the σ° measurement z_k. Furthermore, $\sigma^2_{k,i}$ is the variance of the measurement z_k assuming a wind of $\mathbf{w}_i$.

We want to develop a statistical test to determine if we should retain ambiguity $\mathbf{w}_0$ for further processing. To do this we evaluate the probability that the observed ratio of likelihood values would be as large if $\mathbf{w}_0$ is the true wind. In other words we want to evaluate the probability that $p_{\mathbf{z}}(\mathbf{z}|\mathbf{w}_1) \geq \mathcal{K} p_{\mathbf{z}}(\mathbf{z}|\mathbf{w}_0)$ given that $\mathbf{w}_0$ is the true wind. The constant $\mathcal{K}$ is the observed likelihood ratio, $\mathcal{K} = p_{\mathbf{z}}(\mathbf{z}_0|\mathbf{w}_1)/p_{\mathbf{z}}(\mathbf{z}_0|\mathbf{w}_0)$. In other words, define

$$\alpha_0(\mathcal{K}) = \text{Prob}[p_{\mathbf{z}}(\mathbf{z}|\mathbf{w}_1) \geq \mathcal{K} p_{\mathbf{z}}(\mathbf{z}|\mathbf{w}_0)|\mathbf{w}_0], \qquad (1)$$

α_0 is then the probability that the likelihood ratio, $\mathcal{K}$, would be observed if the lower-ranked ambiguity ($\mathbf{w}_0$) is the true wind. A small value of α_0 suggests that $\mathbf{w}_0$ is not a reasonable wind estimate and can thus be discarded.

To simplify the calculation of α_0 we use the log-likelihood ratio $\log \mathcal{L}(\mathbf{z})$ where

$$\log \mathcal{L}(\mathbf{z}) = \log[p_{\mathbf{z}}(\mathbf{z}|\mathbf{w}_1)/p_{\mathbf{z}}(\mathbf{z}|\mathbf{w}_0)]$$

and calculate the probability that $\log \mathcal{L}(\mathbf{z}) \geq \log \mathcal{K}$ given $\mathbf{w}_0$ is the true wind. This ratio can be written as

$$\log \mathcal{L}(\mathbf{z}) = \sum_{k=1}^{N} \left[\log\left(\frac{\sigma_{k,0}}{\sigma_{k,1}}\right) + \frac{(z_k - M_{k,0})^2}{2\sigma^2_{k,0}} - \frac{(z_k - M_{k,1})^2}{2\sigma^2_{k,1}} \right], \qquad (2)$$

where again $M_{k,i}$ denotes the value of M_k assuming a wind of $\mathbf{w}_i$ and $\sigma_{k,i}$ is the value of σ_k assuming a wind of $\mathbf{w}_i$. Note that in this paper the CMOD-FDP GMF is used [1].

We define several variables useful in simplifying this expression and obtaining the probability density function in order to evaluate the probability.

$$a = \sum_{k=1}^{N} \log\left(\frac{\sigma_{k,0}}{\sigma_{k,1}}\right), \quad b = \sum_{k=1}^{N} \frac{(M_{k,0}-M_{k,1})^2}{2\sigma_{k,1}^2},$$
$$c_k = \frac{1}{2}\left(1 - \frac{\sigma_{k,0}^2}{\sigma_{k,1}^2}\right), \quad d_k = \frac{\sigma_{k,0}(M_{k,0}-M_{k,1})}{\sigma_{k,1}^2},$$
$$x_k = \frac{(z_k) - M_{k,0}}{\sigma_{k,0}},$$

and the random variable l,

$$l = \log \mathcal{L}(\mathbf{z}) - a + b = \sum_{k=1}^{N} \left(c_k x_k^2 - d_k x_k\right). \quad (3)$$

Note, that assuming $\mathbf{w}_0$ is the true wind implies that each x_k is a zero-mean, unit-variance, Gaussian random variable. Now, α_0 can be computed as the probability that l, a random variable which is a function of the measurements, is greater than or equal to $y = \log \mathcal{K} - a + b$ given that $\mathbf{w}_0$ is the true wind. Thus,

$$\alpha_0(\mathcal{K}) = \int_y^{\infty} p_l(l|\mathbf{w}_0)dl. \quad (4)$$

Calculating the probability function of l is a difficult task in general. However, the moment-generating function can be obtained in a fairly straightforward manner since the measurements are independent. The result is

$$\Phi_l(s) = \prod_{k=1}^{N} \frac{1}{\sqrt{1-2c_k s}} \exp\left(\frac{d_k^2 s^2}{2-4c_k s}\right) \quad (5)$$

The central limit theorem guarantees that as N gets large (corresponding to more measurements) the distribution of l approaches a normal distribution with mean $\eta = \sum_{k=1}^{N} c_k$ and variance $\sigma^2 = \sum_{k=1}^{N}(d_k^2 + 2c_k^2)$. For large N this makes calculation of α_0 a trivial task. For small N the distribution of l is more complicated but can be expressed asymptotically as a normal density plus an error term. The error term can be written in terms Hermite polynomials and moments of the density function [2]. These moments can be obtained, in principle, from the moment-generating function of l. Thus, a series solution for α_0 can be found. Defining $v = (y - \eta)/(\sqrt{2}\sigma)$ we obtain.

$$\alpha_0 = \frac{1}{2}\left[1 - \text{erf}(v)\right] + \sum_{k=3}^{\infty} \frac{C_k}{\sqrt{\pi}} e^{-v^2} H_{k-1}(v) \quad (6)$$

where

$$C_k = \sum_{m=0}^{\left[\frac{k-3}{2}\right]} \frac{(-1)^m \left(E\left[(l-\eta)^{k-2m}\right] - L_{k,m}\right)}{m!(k-2m)!\sigma^{k-2m}2^{k/2+m}} \quad (7)$$
$$L_{k,m} = \begin{cases} 0 & k \text{ odd}, \\ \sigma^{k-2m}(k-2m-1)!! & k \text{ even}. \end{cases} \quad (8)$$

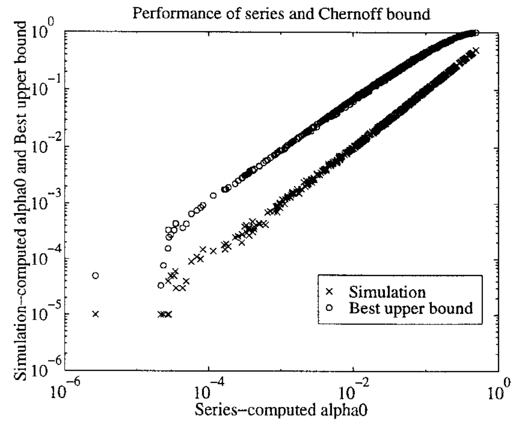


Figure 1: *Plot of α_0 estimated from simulation versus α_0 estimated from four terms of series in (6). Also a plot of the best Chernoff bound versus the series calculation of α_0.*

Simulation suggests that more than four terms are required to get an accurate value of α_0 when there is large variance in the measurements due to large modeling error. Since the moments of l become increasingly cumbersome to calculate, an alternative approach may be used which involves only an easily computable upper bound on the value of α_0.

By establishing an upper bound on α_0, a conservative decision can be based on the value of the upper bound rather than the exact value. A relatively tight upper bound is the Chernoff bound which states that for arbitrary $s > 0$,

$$\alpha_0 \leq e^{-sy}\Phi_l(s) \quad (9)$$

where $\Phi_l(s)$ is the moment generating function of l.

The tightest Chernoff bound is obtained by finding the value of s that minimizes $e^{-sy}\Phi_l(s)$, or $-sy + \log(\Phi_l(s))$. Such a value is a positive real root of a polynomial of order $2N$, restricted to ensure a real value of $\Phi_l(s)$. Thus, the minimum value of the bound can be found in a computationally tractable way. However, to speed computation a single value of s for global use is often used. Testing using ERS-1 orbit parameters shows that $e^{-sy}\Phi_l(s)$ is extremely flat around the minimum value near $s = 1$. Thus, $s = 1$ is a satisfactory choice for obtaining a useful upper bound. Numerically evaluating this bound provides a value to be used in the probability test given in (1). Remembering that α_0 is the probability of discarding an ambiguity which is the true wind, a threshold for this probability is selected. The Chernoff bound ($s = 1$) is then computed. Any ambiguities with probability bounds less than the threshold can be discarded with confidence since the probability that the discarded ambiguity is the true wind is less than the chosen threshold.

Since it is desirable to throw out as many aliases as possible without removing the true wind, it is advantageous to evaluate how tightly the Chernoff bound approaches α_0 for various wind vector cells and wind estimates. Fig. 1 shows the relationship between Monte-Carlo calculation of α_0 and a four-term series calculation using (6) for all of the aliases in 500 wind vector cells. Actual measurements are used.

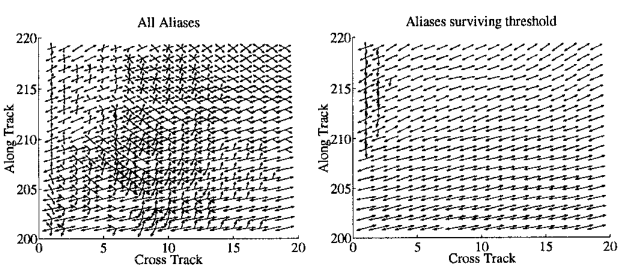
Two distinct relationships are shown in Fig. 1. The first is a plot of simulation-calculated α_0 versus series-calculated α_0 which shows excellent agreement between simulation and series calculations. The second is a plot of the best Chernoff upper bound versus the series calculation of α_0. Note that for a given upper bound the actual value of α_0 for that likelihood ratio is consistently a factor of 10 lower. This implies that the upper bound selected for thresholding can be about 10 times larger than the acceptable probability of throwing out the correct wind. Thus if a 10^{-4} probability of throwing out the correct wind is desired, a threshold of 10^{-3} can be selected.

PERFORMANCE

The utility and performance of the method of eliminating wind aliases described in this paper can be illustrated with the aid of Fig. 2. In part (a), all wind aliases returned by a ML wind retrieval algorithm for a particular region within an ERS-1 swath are kept. In part (b) only the ambiguities greater than a threshold of 10^{-3} are shown. Note how the resulting "most probable ambiguities" field clearly enables determination of the wind streamline.

To further analyze the performance of upper-bound thresholding on removing wind aliases, we use simulated measurements based on known wind fields and ERS-1 orbit parameters. For this simulation only communication noise is added. The results are in summarized in Fig. 3 where a comparison is made between using the thresholding scheme and not using it. When all wind aliases are kept, there is an average of 3.4 aliases per cell. Applying an upper-bound- probability threshold of 0.001 reduces this number to 2.3 aliases per cell with a large majority of the cells having only 2 aliases. This reduction is obtained without ever erroneously discarding the alias closest to the true wind.

As a final demonstration, actual σ° measurements are used from the ascending portion of ERS-1 revolution 7220. A graph similar to Fig. 3 is shown in Fig. 4. As in the simulations, the scheme eliminates all but two aliases for nearly all of the wind vector cells. Since the true wind is unavailable, a count is made of how many times the alias closest to the wind selected by the European Space Agency (ESA) for that cell is discarded.



(a) All aliases

(b) Aliases resulting in upper bound $\leq 10^{-3}$ are removed

Figure 2: *Comparison plot suggesting that eliminating improbable wind aliases can aid ambiguity removal. Data taken ascending part of revolution 7220.*

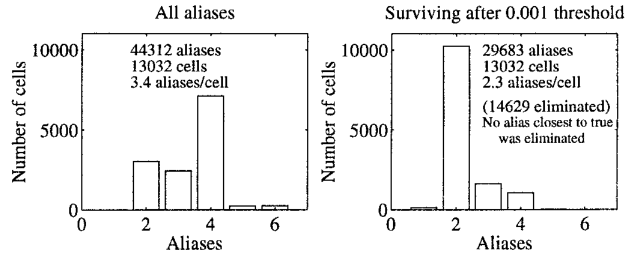


Figure 3: *Histograms of the number of cells with a given number of aliases when thresholding is applied versus when it is not applied using simulated measurements.*

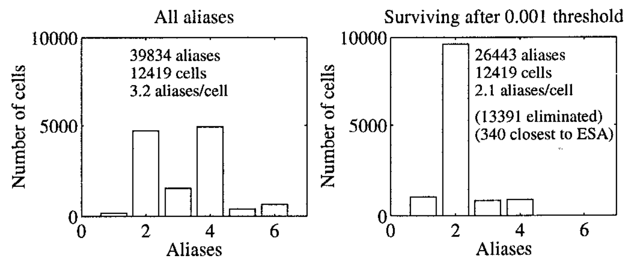


Figure 4: *Histograms of the number of cells with a given number of aliases when thresholding is applied versus when it is not applied using real measurements from ERS-1 revolution 7220 ascending.*

CONCLUSION

We have developed a statistical test which can be used to effectively to distinguish between solutions in inverse problems in which maximum likelihood estimation results in multiple maxima. The technique is applied to eliminating improbable wind solutions arising in point-wise wind estimation using scatterometer data. Evaluation of the technique suggests that it can be used effectively to support ambiguity removal by eliminating all but two wind aliases for nearly all of the wind vector cells in a swath.

REFERENCES

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