Wind Measurement Resolution for a Scanning Pencil Beam Scatterometer

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Abstract - The problem of determining the effective wind measurement resolution of a wind scatterometer is considered. Using simplifying assumptions and approximations, a simple linear model for wind retrieval is derived and related to the effective wind estimate resolution and the backscatter measurement resolution. This enables an analytic evaluation of the scatterometer wind measurement resolution. The effective wind resolution is shown to be related to the unique area of the σ^o measurements used to retrieve the wind. This area is a function of the σ^o measurement cell sizes and their coregistration. Using this result the resolution of the *SeaWinds* scatterometer is analyzed.

I. INTRODUCTION

One of the key requirements for the design of a scatterometer system is the resolution of the wind measurements. However, determining the actual wind measurement has been difficult due to the nonlinear retrieval algorithms used to infer the wind from the backscatter measurements. In particular, the actual wind measurement resolution may depend on the true wind field.

Wind scatterometers measure the radar backscatter from the ocean's surface. Using measurements of the backscatter of an area on the ocean's surface taken from several azimuth angles, the wind is retrieved or estimated using a geophysical model function relating the wind vector and the radar backscatter. The individual σ^o measurements used to retrieve the wind have differing sizes and orientation and may be spatially offset from each other. The difficulty in determining the effective resolution is suggested by Fig. 1 which illustrates two different cases of σ^o cell coregistration.

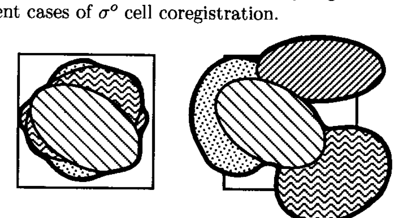

Figure 1.
Overlapping σ^o cells (shaded areas) with different coregistrations. The unique area is outlined with a heavy line.

In this paper, a simplified formulation for the wind measurement resolution is developed which does not depend on the true wind field. It is shown that the resolution of the estimated wind is determined by the "unique area" of the backscatter measurements combined to estimate the wind. The unique area is the region covered at least once by a backscatter measurement (see Fig. 1). The analysis was specifically developed to determine the resolution of a scanning pencil bean antenna system but can be applied to virtually any scatterometer system. The analysis considers the backscatter resolution element size and coregistration accuracy and has proven useful in making instrument system tradeoffs in developing a new scatterometer design known as *SeaWinds* which is described in separate papers (Freilich et al., 1994; Wu et al., 1994).

II. MEASUREMENT RESOLUTION

An ideal measurement consists of a perfectly accurate sample at an infinitely small point of the desired field. However, "real" measurements must be made over a finite area. Associated with the measurement area is a measurement response denoted by $h(x,y)$. For simplicity we will consider the one-dimensional case, though the results can be extended to multiple dimensions.

The n^{th} measurement z_n may be written as the integral of the field $s(x)$ to be measured (σ^o) with the instrument response for the n^{th} observation, i.e.,

$$z_n = \int h_n(x)s(x)dx. \tag{1}$$

In application, we make the measurements z_n over a (nearly) uniform sampling grid with sample spacing T. (Typically, $T = 50$ or $T = 25$ km in wind scatterometry.) Assuming that the instrument measurement response is the same at each sample point for simplicity, the measurements may be considered to be discrete samples (at $x = nT$) of a continuous function $f_s(x)$ where

$$f_s(x) = \int h(\tau - x)s(\tau)d\tau = h(x) * s(x) \tag{2}$$

where "*" denotes linear convolution.

While alternate definitions can be used, a useful definition of resolution is the highest maximum spatial frequency ("wavenumber") which can be accurately measured. In the wavenumber domain Eq. (2) becomes

$$F_s(\omega) = H(\omega)S(\omega) \tag{3}$$

where $H(\omega)$ and $S(\omega)$ are the Fourier transforms of $h(x)$ and $s(x)$, respectively. We see that $H(\omega)$ acts as a low pass filter on the spectrum of $F_s(\omega)$ and may introduce spectral nulls. The highest wavenumber which can be measured is determined by the bandwidth of $H(\omega)$. In general, $F_s(\omega)$ is only a good estimate of $S(\omega)$ when $H(\omega)$ has a very broad main lobe, i.e., when $h(x)$ is narrow. In addition, note that the discrete sampling of $f_s(x)$ produces spectral aliasing if $S(\omega)$ is not bandlimited.

III. THE WIND SPECTRUM AND ALIASING

The atmosphere exhibits scales of motion ranging from planetary to molecular. Thus, there is energy at all possible wavenumbers and $S(x)$ is not bandlimited. This implies at least some aliasing in the wind measurements; however, there is much less energy at larger wavenumbers (high "frequencies" or small scales) than at small wavenumbers (low "frequencies" or large scales). Note that because the wind spectrum has energy at all wavenumbers, NO scatterometer system can provide sufficient resolution to avoid aliasing.

Freilich and Chelton (1986) found that the one-dimensional energy spectra of the wind $[V(\omega)]$ rolls off as the inverse of the wavenumber squared, i.e.,

$$V(\omega) \propto \omega^{-2}. \tag{4}$$

If we assume that the spectral shape and rolloff is similar on smaller scales, this spectral model can be used to evaluate the expected aliasing levels in the scatterometer measurements.

Assume that the wind is sampled on a uniform grid with spacing T. It can be shown that the ratio of the "single-fold" aliased wind to the desired wind at a wavenumber of $2\pi/\lambda$ (where $\lambda > T$) is $(1 - \lambda/T)^2$. A plot of this ratio as a function of λ for $T = 50$ km is shown in Fig. 2. Note that the "aliasing" ratio is < -20 dB for $\lambda \geq 100$ km, twice the sample spacing. At this level aliasing will not be a problem. This is fortunate since 100 km is the smallest spatial scale which can be resolved with 50 km sampling. We conclude that for spatial scales larger than twice the sample spacing, aliasing can be ignored.

IV. RESOLUTION OF A LINEAR ESTIMATOR

In wind scatterometry several measurements are combined (via wind retrieval which is non-linear) to estimate the wind. The question arises:

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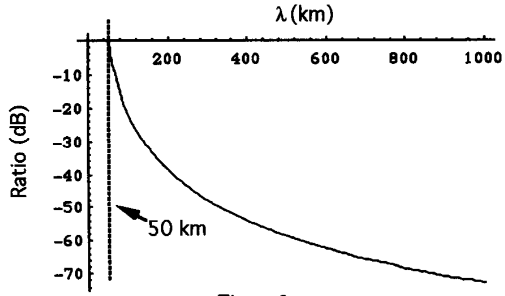

Figure 2.
Aliasing ratio vs. spatial scale (λ).

What is the effective resolution of the wind estimate? We first consider an estimator $\widehat{U}$ consisting of a linear combination of the measurements z_n,

$$\widehat{U} = \sum_n a_n z_n \tag{5}$$

where a_n are constants. Substituting from Eq. (1),

$$\begin{aligned} \widehat{U} &= \sum_n a_n \int h_n(x)s(x)dx = \int \left[\sum_n a_n h_n(x) \right] s(x)dx \\ &= \int q(x)s(x)dx. \end{aligned} \tag{6}$$

Thus, an estimator consisting of a linear sum of measurements can be expressed in the form of Eq. (1) with $h(x)$ replaced with $q(x)$ where $q(x)$ is a weighted sum of the individual measurement responses. In general, the effective resolution of the estimator will be inferior to the measurements.

As a graphical example, consider the three measurement responses schematically illustrated in Fig. 3. Figure 3b shows the effective response function $q(x)$. The resulting $q(x)$ is wider than the individual $h_n(x)$ so that resolution of the estimator is always less than an individual measurement. Since $q(x)$ defines the effective resolution of the linear estimate, the resolution will be dependent on the individual measurement resolutions *and* how well they are "coregistered." An approximate estimate of the resolution is given by the indicator function $iq(x)$ which delineates the unique area. $iq(x)$ provides a lower bound on the resolution. While the actual resolution is given by the wavenumber bandwidth of $q(x)$, $iq(x)$ provides a useful measure of the resolution and will be used in the sequel.

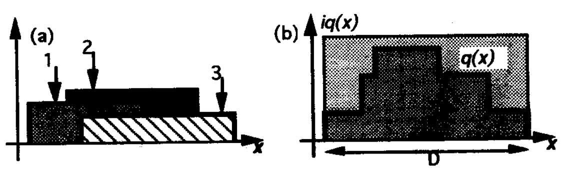

Figure 3.
Measurement resolution using a linear estimator.

V. WIND MEASUREMENT RESOLUTION

While the resolution of a linear estimator is given by the unique area of the measurements, wind retrieval is a non-linear process. However, by making some approximations a simple linear model for the wind speed estimate can be derived.

To begin we write the n^{th} (noise-free) observation of σ^o as

$$\sigma_n^o = \mathcal{M}_n(\vec{U}) \approx G_n U^{H_n} \tag{7}$$

where $\vec{U}$ is the true wind vector, U is the magnitude (speed) of $\vec{U}$, and $\mathcal{M}_n(\cdot)$ denotes the geophysical model function relating σ^o and the wind. The subscript n on $\mathcal{M}_n(\cdot)$ subsumes the incidence and azimuth angle (as well as polarization) dependence of the model function. G_n and H_n are the coefficients a "G-H" table form of the model function.

To simplify the problem we ignore the azimuth dependence of σ^o, and estimate only the wind speed U. Noting that $H_n \sim 2$,

$$\begin{aligned} z_n &= \int h_n(x)\sigma_n^o(x)dx = \int h_n(x)G_n[U(x)]^{H_n}dx \\ &\approx G_n \int h_n(x)U^2(x)dx. \end{aligned} \tag{8}$$

A. Simplified Wind Retrieval Analysis

While the actual wind retrieval is based on Maximum-Likelihood Estimation (MLE), a more tractable analysis results from a least-squares estimator, $\mathcal{J}(U)$,

$$\mathcal{J}(U) = \sum_n [z_n - \mathcal{M}(U)]^2 \tag{9}$$

The value of U which minimizes $\mathcal{J}(U)$ is the wind speed estimate. Taking the derivative of $\mathcal{J}(U)$ (again noting that $H_n \sim 2$)

$$\begin{aligned} \frac{d}{dU}\mathcal{J}(U) &\approx 2U \ln U \left(\sum_n G_n H_n z_n - \sum_n G_n^2 H_n U \right) \\ &= 2U \ln U(\alpha - \beta U). \end{aligned} \tag{10}$$

Setting this to zero and solving for U, the wind speed estimate is,

$$\widehat{U} = \frac{\alpha \sum_n G_n H_n}{\beta \sum_n G_n^2 H_n} z_n = \sum_n b_n z_n. \tag{11}$$

Thus, the wind speed estimate is a linear combination of the measurements z_n. It follows that *the effective resolution of the wind speed estimate is the unique area defined by the overlapped σ^o measurements* and not by just an individual σ^o cell's resolution. Thus, both the σ^o cell size *and* the coregistration of the σ^o cells are crucial to achieving high wind estimate resolution. While a simplistic analysis, this result suggests that the wind *vector* estimate resolution is also given by the unique area of the σ^o measurements.

B. Spatial Sampling Criteria

Let us now consider the relationship of the sample spacing T and $q(x)$. Figure 4 illustrates the filtering effect of different $q(x)$ on the estimated wind spectrum [given by Eq. (4)] when the sample spacing T is 50 km. For this example $q(x)$ is assumed to be a "boxcar" indicator function with width τ, the effective resolution. We note that for $T/\tau = 1/2$ (when the unique σ^o measurement area is one half of the sample spacing), the filtering effect on the spectrum is negligible. At $T = \tau$ when the sample spacing and measurement area are the same, the filtering effect is significant only for spatial scales smaller than $\sim$ 100 km or twice the sample spacing. However, for $T/\tau = 2$ (when the measurement extent is twice the sample spacing) we see the introduction of a spectral null at 100 km with significant filtering (spectrum attenuation) at larger scales.

Figure 5 presents plots of the ratio of the actual to filtered spectra versus the effective measurement resolution τ for several different spatial scales. The sample spacing is $T = 50$ km in this plot. These figures suggest that τ should be kept similar to T to avoid significant spectral attenuation.

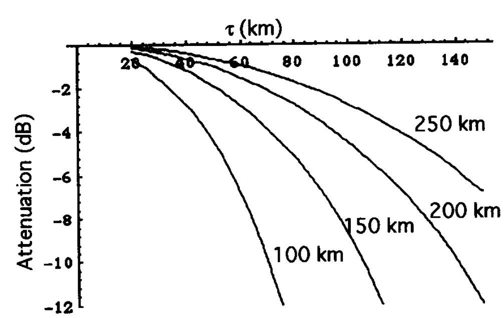

Figure 5.
Attenuation versus τ for $T = 50$ km at different spatial scales.

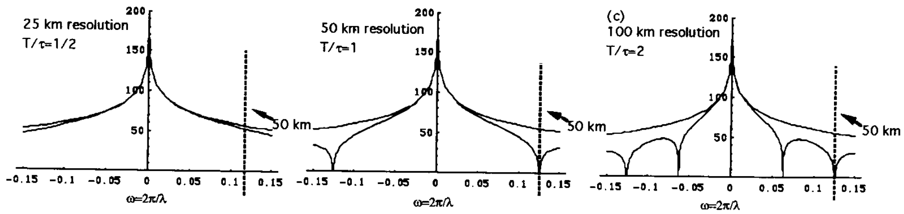

Figure 4.
Actual (upper line) and filtered (lower line) wind spectrum versus wavenumber for various wind cell resolutions with $T = 50$ km spatial sampling. The vertical scale is arbitrary.
Wind cell resolution: (a) 25 km. (b) 50 km. (c) 100 km.

VI. *SeaWinds* RESOLUTION

These results have been applied to the analysis of the resolution of the *SeaWinds* scatterometer, a dual pencil-beam scatterometer being developed for flight on ADEOS-II in 1999. *SeaWinds* is described in detail in separate papers (Freilich et al. 1994; Wu et al. 1994) The baseline *SeaWinds* antenna geometry is given in Table 1. Given the antenna geometry and beamwidths, the orbit, the antenna rotation rate (20 rpm), and the pulse repetition rate (200 Hz, interleaved), the size and locations of the measurements on the Earth's surface can be determined. These are listed in Table 2 while Fig. 6 graphically illustrates the measurement locations.

TABLE 1. *SeaWinds antenna beam parameters.*

Beam	Look Angle (°)	Antenna Beamwidth (°)	
		Azimuth	Elevation
Inner	40	1.8	1.6
Outer	46	1.8	1.3

TABLE 2. *Nominal σ^o cell sizes for SeaWinds* .

beam	incidence angle (°)	root area (km)	size (km)
inner	46.3	34.7	34.6 × 44.5
outer	54.0	38.5	39.2 × 48.3

To retrieve the wind on a rectalinear grid, each of the σ^o cells are assigned ("binnned") to the grid element which contains the cell center. Because of the way the σ^o measurements are made with the scanning antenna, the cells are not aligned with the rectalinear grid, resulting in large cell coregistration differences.

To estimate the wind measurement resolution, the unique area of the σ^o measurements for each grid element were computed. This was done for several possible grid sizes. Averaged over the swath, the effective spatial resolution τ is the square root of the unique area. Table 3 presents the results for the baseline *SeaWinds* design. While the resolution of the σ^o measurements is $\sim$ 45 km, the wind estimate resolution depends on the grid size selected. For the nominal $T = 50$ km grid, the effective resolution is 78 km which meets the 100 km requirement for *SeaWinds*.

TABLE 3. *SeaWinds resolution versus grid size.*

	Grid spacing (km)			
	25	30	50	60
Nominal Resolution (km)	50	60	78	85

VII. SUMMARY

We have shown that the effective wind estimate resolution is given by the weighted sum of the measurement responses of the σ^o cells used to retrieve the wind. This can be reasonably approximated by the unique area of the σ^o measurements. The unique area depends on both the size of the individual σ^o measurements and their coregistration. This permits a tradeoff between cell size and the cell coregistration accuracy.

REFERENCES

Freilich, H.H., and D.B. Chelton, "Wavenumber Spectra of Pacific Winds Measured by the Seasat Scatterometer," *J. Phys. Oceanography*, Vol. 16, No. 4, 1986: 741-757.

Freilich, M.H, D.G. Long, and M.W. Spencer, "*SeaWinds*: A Scanning Scatterometer for ADEOS II - Science Overview," in IGARSS Digest, 1994.

Wu, C., J. Graf, M. Freilich, D. Long, M. Spencer, W. Tsai, C. Winn, "The *SeaWinds* Scatterometer Instrument," in IGARSS Digest, 1994.

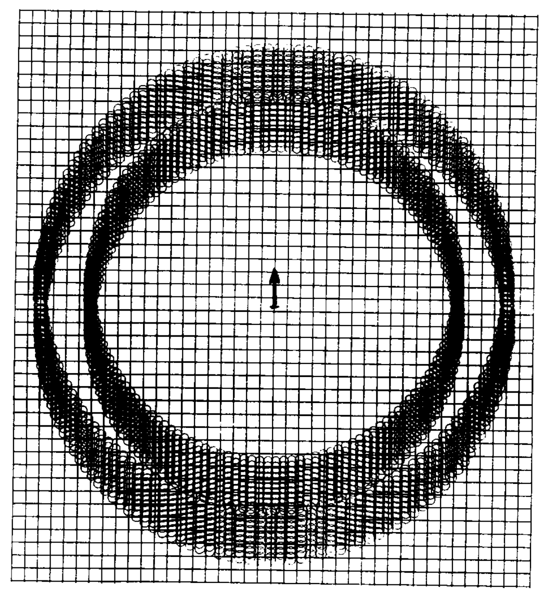

Figure 6.
SeaWinds σ^o measurement layout on the Earth's surface for several rotations of its antenna beams with a 50 km grid.