An Improved Simulation Model for Spaceborne Scatterometer Measurements

Peter K. Yoho, *Student Member, IEEE*, and David G. Long, *Senior Member, IEEE*

***Abstract*—Development of scatterometer designs and applications requires extensive data simulation. The advancing capabilities of instruments motivates our proposal for an improved simulation model for noisy scatterometer measurements. Previous simulation models do not separately account for the two forms of random variation—signal fading and additive noise—which affect scatterometer measurements. The proposed model is able to generate data that are statistically equivalent (in a mean and variance sense) to actual instrument measurements by accounting for both variations, while maintaining ease of implementation. The model is particularly adept at handling design tradeoffs related to signal-to-noise ratios by appropriately separating fading and additive noise. Unlike previous models, the new model also can account for correlation between measurements, an issue that has recently become important.**

***Index Terms*—Data simulation, scatterometry.**

I. INTRODUCTION

DEVELOPMENT of scatterometer instruments and data processing algorithms relies heavily on simulation. In particular, modeling and simulation of random fluctuations in the measurements is a critical portion of assessing the performance of a design. For scatterometry, variations occur in two forms. The first is fading due to the coherent summation of responses from multiple surface scatterers. Fading is multiplicative in nature and is related to the magnitude of the surface response. The second random variation is additive noise caused by a variety of thermal and environmental factors [1].

Previous simulation models couple the two variation sources, fading and noise, into one equivalent term, e.g., [2]. While this assumption may be appropriate in some circumstances, developing scatterometer applications benefit from improved simulation models. In particular, the two sources of variation, while independent in origin, are correlated by the nonlinear square law processing used by most instruments. Further, proper definition and determination of signal-to-noise ratios (SNRs) is difficult when using a single variation term. Complex models that properly account for these variations have been created for some instruments. However, they are instrument specific and extremely time consuming to develop. This limits their utility in simulations designed to use data from multiple instruments.

To facilitate development of new instrument designs and additional scatterometer data applications, we propose an improved statistical model for simulating scatterometer data. The proposed model is specific enough, by accounting for fading and noise separately, to accurately simulate data variations, yet general enough to be quickly and easily adapted to any microwave scatterometer configuration. Example applications for the model include development of sample backscatter fields for wind estimation, image enhancement, and polar climate studies [3]–[5].

The presentation of the model is organized as follows. Section II develops a general framework of scatterometer measurements. It discusses causes and effects of variation in a general measurement formulation. Section III then presents the statistical simulation model, applying it to the measurement scheme of Section II as well as externally derived results. Finally, Section IV summarizes findings and concludes.

Manuscript received July 2, 2003; revised August 9, 2003. This work was supported by the National Aeronautics and Space Administration Jet Propulsion Laboratory.

The authors are with the Department of Electrical and Computer Engineering, Brigham Young University, Provo, UT 84602 USA (e-mail:long@ee.byu.edu)

Digital Object Identifier 10.1109/TGRS.2003.818340

II. DATA COLLECTION

Scatterometers operate by transmitting pulses of microwave energy and measuring the amount of energy that returns to the instrument. The measurement of returning backscatter is directly related to the normalized radar cross section (σ°) of the surface [1]. By assuming a large number of individual scatterers in the measurement and that no one scatterer dominates, the voltage response of each measurement Z can be modeled as a complex random variable having a Rayleigh distributed magnitude and a uniformly distributed phase [1]. The random nature of each pulse is caused by the coherent integration of all scatterers illuminated. σ° is related to the voltage response through the expected value squared

$$A_e\sigma^\circ = \mathcal{E}\left[|Z|^2\right] \tag{1}$$

where $\mathcal{E}$ is the expected value operator, and A_e is the area of instrument illumination. From σ° values, physical parameters such as surface roughness and near surface wind can then be inferred [1], [6].

Unfortunately, the received signal is corrupted by additive noise. Noise comes from multiple sources, most notably from internal thermal excitement of the instrument electronics and radiometric radiation incident on the instrument antenna. Since the magnitude of the internal thermal noise is the most significant, we model all additive variation as coming from this source. Thermal noise in the receiver bandwidth is zero mean and normally distributed with a white spectrum.

A variety of instrument measurement configurations have been used for past instruments [6]–[10]. For this letter, we develop our simulation model using the general measurement

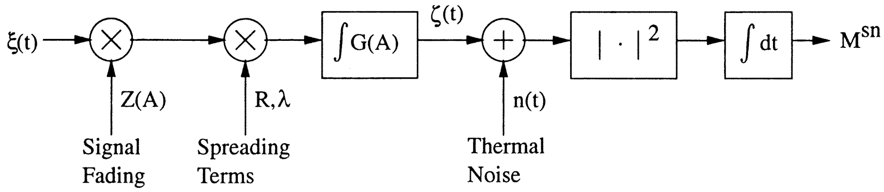

Fig. 1. Simplified signal flow model for scatterometer measurements. See text for definition of terms.

scheme used in [7] and described by Fig. 1. We then demonstrate application of the model to other measurement forms.

A transmitted microwave signal $\xi(t)$ scatters off the surface having response Z. In the course of flight, the signal spreads as a function of instrument-to-surface range r and wavelength, λ. The received response is integrated over its area A with weighting equivalent to the gain of the antenna G. Thermal noise $n(t)$ is modeled internal to the instrument by placing it after the antenna gain filtering. In the final step of processing, the received signal is squared to create a power signal and integrated to obtain the total measurement energy.

Mathematically, we can define the transmitted signal as

$$\xi(t) = \sqrt{E_t}a(t) \tag{2}$$

where E_t is the total transmitted energy, and $a(t)$ is the pulse modulation function. The received signal M^{sn} can then be written as

$$M^{sn} = \int_{T_s} \left| \left[\int_A \frac{\xi(t)Z(A)\lambda G(A)}{(4\pi)^{3/2}r^2(A)} \, \mathrm{d}A \right] + n(t) \right|^2 \mathrm{d}t \tag{3}$$

$$= \int_{T_s} |\zeta(t) + n(t)|^2 \, \mathrm{d}t \tag{4}$$

where $\zeta(t)$ denotes the signal portion of the measurement (enclosed in brackets in (3) and shown in Fig. 1). By using the bulk range $\bar{r}$ in the denominator, assuming that all filters are ideal, and σ° is constant over the area of integration, and defining the variance of the noise as $n_o/2$, it can be shown that [9]

$$\mathcal{E}[M^{sn}] = X\sigma^\circ H + 2B_sT_s\frac{n_o}{2} \tag{5}$$

where

$$X = \frac{E_t\lambda^2G_o^2A_E}{(4\pi)^3\bar{r}^4} \tag{6}$$

with G_o as the peak antenna gain, A_E the effective area of the measurement

$$A_E = \frac{1}{G_o^2}\int\limits_A G^2(A)A_e(A)\mathrm{d}A \tag{7}$$

and

$$H = \frac{1}{G_o^2A_E}\int\limits_{T_s}\int\limits_A G^2(A)\left|a\left(\frac{t-2r(A)}{c}\right)\right|^2 A_e(A)\,\mathrm{d}A\,\mathrm{d}t. \tag{8}$$

The receiver bandwidth B_s and the time of integration T_s are assumed wide enough to fully capture the signal.

Scatterometers remove the bias caused by the noise by making a separate noise-only measurement M^n, so that [1]

$$M^u = \varsigma M^{sn} + \varphi M^n \tag{9}$$

where M^u is the unbiased measurement. Given that $\mathcal{E}[M^n] = 2B_nT_nn_o/2$, where B_n and T_n are the noise-only bandwidth and integration time, ς and φ are defined ($\varsigma = H^{-1}$, $\varphi = -B_sT_s/HB_nT_n$) so that $\mathcal{E}[M^u] = X\sigma^\circ$.

III. Modeling Scatterometer Measurement Variation

A commonly used metric to express measurement variability is K_p, the normalized standard deviation of the measurements, defined as

$$K_p = \frac{\sqrt{\mathrm{Var}[\sigma^\circ{}_{\mathrm{meas}}]}}{\mathcal{E}[\sigma^\circ]}. \tag{10}$$

A generally used form of K_p is

$$K_p = \alpha(\sigma^\circ)^2 + \beta(\sigma^\circ) + \gamma \tag{11}$$

where α is related to signal variations, γ is related to noise variations, and β is a cross-correlation term [2], [3], [10]. The α, β, and γ coefficients depend on the instrument design and measurement SNR.

Past simulation models have simulated noisy σ° measurements using

$$z = X\sigma^\circ(1 + K_p\nu) \tag{12}$$

where ν is a zero-mean Gaussian random variable [2]. An estimate of σ° is then

$$\hat{\sigma}^\circ = \frac{z}{X}. \tag{13}$$

This model, while simple and widely used, lumps the variations of both the multiplicative signal fading and the additive noise into one variable: ν. For some applications, this model is adequate; however, it is difficult to separate the two independent variations and, thus, handle SNR design tradeoffs and measurement correlation effects.

Our proposed model accounts for these issues by modeling a scatterometer measurement as the sum of the mean backscatter $m = X\sigma^\circ$ and two random variables

$$z = m + Ax + By \tag{14}$$

where A and B are instrument specific coefficients, and x and y are zero mean, unit variance, Gaussian-distributed random variables. The two random variables x and y have a correlation coefficient

$$\mathcal{E}[xy] = \rho \tag{15}$$

where $0 \leq \rho \leq 1$. While the origins of $\zeta(t)$ and $n(t)$ are independent, the power-law squaring procedure used in processing correlates the signals. The ρ term accounts for this.

We note that modeling of the signal fluctuation x as Gaussian may not always be a valid assumption. Scatterometers integrate the squared signal over a particular time and bandwidth. Application of the central limit theorem suggests that the Gaussian approximation is valid when the time bandwidth product approaches 10 or greater [1]. For most previous instruments, this product has been in the hundreds, easily justifying the assumption, though there have been some exceptions due to the processing used [11]. Newer instruments use high pulse rates that can lower the time bandwidth product to levels where this assumption becomes questionable. In this case, the distribution of x can be modified appropriately.

To adapt the model to a particular instrument, values for A, B, and ρ must be determined. While expressions for K_p have been reported for several instruments, we present only two examples here. In general, the A term accounts for the variance of the measurements due to fading, and the B term accounts for the noise in the signal, both in M^{sn} and M^n. Since x and y are zero mean, the expected value of the model measurement is $\mathcal{E}[z] = m$.

For the measurement scheme developed in [7], the variance of an unbiased measurement has the form of (11) and can be expressed as

$$\text{Var}[M^u] = \left(\frac{X\sigma^\circ}{H}\right)^2 \left(V + SU + 2S^2[I(2T_sB_s) + I(2T_nB_n)]\right) \tag{16}$$

where V is the normalized signal variance, S is the SNR, U is the signal-noise cross-correlation function, and I is a special function that starts at 1 ($I(0) = 1$) and converges to the inverse of its arguments as it increases (see [7]).

Our simulation model can be adapted to this scenario by choosing

$$A = \frac{X\sigma^\circ\sqrt{V}}{H} \tag{17}$$

and

$$B = \left(\frac{X\sigma^\circ S}{H}\right)\sqrt{2[I(2T_sB_s) + I(2T_nB_n)]} \tag{18}$$

with

$$\rho = \frac{U}{2\sqrt{2V[I(2T_sB_s) + I(2T_nB_n)]}}. \tag{19}$$

It should be noted, that while the B term in (18) appears to be dependent upon the return signal magnitude, the noise-to-signal term S

$$S = \frac{1}{X\sigma^\circ}\left(2T_sB_s\frac{n_o}{2}\right) \tag{20}$$

cancels out the signal values, resulting in B being dependent only on the noise (18), i.e.,

$$B = \left(\frac{\frac{2T_sB_sn_o}{2}}{H}\right)\sqrt{2[I(2T_sB_s) + I(2T_nB_n)]}. \tag{21}$$

Spencer *et al.* [8] derived the basic statistics for the SeaWinds instrument, showing the variance (in simplified form) to be

$$\text{Var}[M^u] = (X\sigma^\circ)^2\left(\frac{1}{BT_s} + \frac{2S}{BT_n} + \frac{S^2}{BT_n}\right) \tag{22}$$

where B is the bandwidth of both the signal and noise measurements. This variance form is similar to that of the Active Microwave Instrument (AMI) on the European Remote Sensing 1 (ERS-1) satellite [12]. The model is adapted to [8] using

$$A = \frac{X\sigma^\circ}{\sqrt{BT_s}} \tag{23}$$

$$B = \frac{X\sigma^\circ S}{\sqrt{BT_n}} = \frac{BT_sn_o}{\sqrt{BT_n}} \tag{24}$$

$$\rho = \sqrt{\frac{T_s}{T_n}}. \tag{25}$$

In both adaptations, it can be seen that ρ is nonnegative, since all of the terms are positive. Additionally, in [7] $U^2 \approx 4V/B_sT_s$, allowing ρ to satisfy its definition. The second adaptation [8] also satisfies $\rho \leq 1$, since $T_n > T_s$ for SeaWinds.

The separation of signal and noise components in the model allows for significant simplification in the control of simulated SNRs. In both cases illustrated, the A parameter depends only upon the signal, and the B parameter depends only upon the noise. The SNR is, thus, the ratio of A to B scaled appropriately to account for the other terms. Similarly, $S \propto B/A$.

A final consideration in the development of the statistical model is its simulation of correlation between measurements. This topic has previously been irrelevant due to small measurement overlap and low sampling rates of instruments. Recent designs that oversample the surface now require its consideration. Using the results of Yoho and Long [9], the correlation between measurements (9) can be shown to be

$$\begin{aligned}\mathcal{E}[M_k^uM_l^u] =& \frac{X_kX_l\sigma^\circ{}_k\sigma^\circ{}_l}{H_kH_l}\Big(H_kH_l + W + \delta(k-l)\\ &\cdot\left[SU + 2S^2(I(2B_sT_s) + I(2B_nT_b))\right]\Big)\end{aligned} \tag{26}$$

where W is the fading covariance between the two measurements. When $k = l$, $W = V$. Comparing this to our model, we obtain

$$\begin{aligned}\mathcal{E}[z_kz_l] =& \mathcal{E}[(m_k + A_kx_k + B_ky_k)(m_l + A_lx_l + B_ly_l)]\\ =& m_km_l + A_kA_l\mathcal{E}[x_kx_l] + A_kB_l\mathcal{E}[x_ky_l]\\ &+ B_kA_l\mathcal{E}[y_kx_l] + B_kB_l\mathcal{E}[y_ky_l],\end{aligned} \tag{27}$$

which requires additional consideration for the two random terms, x and y. First, we choose y to be independent, identically distributed so that $\mathcal{E}[y_ky_l] = \delta(k-l)$. This models the behavior of thermal noise that is independent from one pulse to another. Conversely, the fading between pulses is correlated with

$$\mathcal{E}[x_k x_l] = \frac{W}{\sqrt{V_k V_l}}. \tag{28}$$

The correlation expression is then

$$\mathcal{E}[z_k z_l] = m_k m_l + \left(\frac{W}{\sqrt{V_k V_l}}\right) A_k A_l + \delta(k-l) B_k B_l + \rho_{kl} A_k B_l + \rho_{lk} A_l B_k \tag{29}$$

where

$$\rho_{kl} = \mathcal{E}[x_k y_l] \tag{30}$$

is the cross-correlation coefficient. Assuming that both measurements, x_k and x_l, have an identical time-bandwidth product, ρ_{kl} for the scheme in [7] is

$$\rho_{kl} = \delta(k-l) \frac{U}{2\sqrt{2V[I(2T_s B_s) + I(2T_n B_n)]}} \tag{31}$$

which is a consistent extension of the single measurement definition of ρ.

IV. Summary and Application

Simulation of variations in scatterometer measurements is an essential part of performance assessment of data applications. The proposed model simply and accurately produces simulated measurements for these purposes. The model can be easily adapted to most existing instruments and is capable of handling measurement correlation in developing instruments. By separately handling the two components in the measurement variance the model accurately describes their effects and better describes the signal-to-noise performance of a particular algorithm or application. While the model does not simulate every nuance of a particular instrument, it generates values which have statistically equivalent means, variances, and covariances and, therefore, is useful in application development and simulation.

References

[1] F. Ulaby, R. Moore, and A. Fung, *Microwave Remote Sensing: Active and Passive*. Norwood, MA: Artech House, 1986, vol. 2.

[2] D. G. Long and J. M. Mendel, "Identifiability in wind estimation from scatterometer measurements," *IEEE Trans. Geosci. Remote Sensing*, vol. 29, pp. 268–274, Mar. 1991.

[3] C.-Y. Chi and F. K. Li, "A comparative study of several wind estimation algorithms for spaceborne scatterometers," *IEEE Trans. Geosci. Remote Sensing*, vol. 26, pp. 115–121, Mar. 1988.

[4] D. S. Early and D. G. Long, "Image reconstruction and enhanced resolution imaging from irregular sampling," *IEEE Trans. Geosci. Remote Sensing*, vol. 39, pp. 291–302, Feb. 2001.

[5] M. R. Drinkwater and C. C. Ling, "Introduction to the special section on emerging scatterometer applications," *IEEE Trans. Geosci. Remote Sensing*, vol. 38, pp. 1763–1764, July 2000.

[6] F. M. Naderi, M. H. Freilich, and D. G. Long, "Spaceborne radar measurement of wind velocity over the ocean—An overview of the NSCAT scatterometer system," *Proc. IEEE*, vol. 79, pp. 850–866, June 1991.

[7] D. G. Long and M. W. Spencer, "Radar backscatter measurement accuracy for a spaceborne pencil-beam wind scatterometer with transmit modulation," *IEEE Trans. Geosci. Remote Sensing*, vol. 35, pp. 102–114, Jan. 1997.

[8] M. W. Spencer, C. Wu, and D. G. Long, "Improved resolution backscatter measurements with the seawinds pencil-beam scatterometer," *IEEE Trans. Geosci. Remote Sensing*, vol. 38, pp. 89–104, Jan. 2000.

[9] P. K. Yoho and D. G. Long, "Correlation and covariance of satellite scatterometer measurements," *IEEE Trans. Geosci. Remote Sensing*, 2003, submitted for publication.

[10] R. Fisher, "Standard deviation of scatterometer measurements from space," *IEEE Trans. Geosci. Electron.*, vol. GE-10, Apr. 1972.

[11] P. E. Johnson, "Uncertainties in oceanic microwave remote sensing: The radar footprint, the wind–backscatter relationship, and the measurement probability density function," Ph.D., Brigham Young Univ., Provo, UT, 1999.

[12] P. L. Frison and E. Mougin, "Use of ERS-1 wind scatterometer data over land surfaces," *IEEE Trans. Geosci. Remote Sensing*, vol. 34, pp. 550–560, Mar. 1996.