# IMPROVING ELECTROMAGNETIC BIAS ESTIMATES

by

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### ABSTRACT

### IMPROVING ELECTROMAGNETIC BIAS ESTIMATES

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Doctor of Philosophy

The electromagnetic (EM) bias is the largest source of error in the TOPEX/Poseidon and Jason-1 satellite sea surface height (SSH) estimates. Due to incomplete understanding of the physical processes which cause the bias, current operational models are based on empirical relationships between the bias wind speed and significant wave height. These models reduce RMS estimation errors of the EM bias to approximately 4 cm.

To improve EM bias estimation the correlation between the bias and RMS long wave slope is studies using data from tower-based experiments in the Gulf of Mexico and Bass Straight, Australia. Models based on significant wave height and RMS slope are more accurate than models based on wave height and wind speed by at least 50% in RMS error between predicted and ground truth bias values.

Nonparametric models have been proposed as a method to reduce the variability of EM bias estimates. Using tower data, nonparametric models developed from wind speed and significant wave height measurements are shown to provide some improvement over parametric models. It is also shown that the historical discrepancy between satellite and tower EM bias measurements is reduced by nonparametric modeling.

A validity study of rough surface scattering models is conducted for surfaces with Gaussian and power law power spectra. Models in the study include physical optics (PO), geometrical optics, small perturbation method, and small slope approximation. Due to the prevalence of the PO approximation, particular emphasis is placed on the development of a validity criterion for the PO model. An empirical study of the PO approximation shows that the validity of the model is more accurately described by the RMS wave slope than the classic surface curvature criterion for surfaces with a Gaussian power spectrum. For surfaces with a power law PSD, the accuracy of the PO approximation is related to the significant slope (RMS surface height/wavelength of the dominant spectral peak). The validity of other models in the study are also shown to be well approximated by bounds on surface slope.

An EM bias model is derived using the physical optics scattering model, hydrodynamic modulation, and non-Gaussian long wave surface statistics. Using a modulation transfer function, the hydrodynamic modulation of small wave heights is shown to be linearly related to the long wave RMS slope. The resulting EM bias model expresses the relative bias as a function of the long wave surface parameters RMS wave slope, surface skewness, and tilt modulation. Coefficients of the long wave parameters are determined by the short ocean waves, and provide insight into the physical mechanisms that cause the bias. From measured values of the ocean surface profile, estimated values of the bias are computed from the bias model. A comparison of these estimated values with *in situ* EM bias measurements shows a strong correlation between the estimated and measured values.

Nadir and off-nadir measurements of the EM bias collected during the BYU Off-Nadir Experiment (Y-ONE) are presented. The *in situ* measurements are compared with bias estimates computed from an off-nadir generalization of the nadir EM bias model. From theoretical and experimental bias measurements a model of the angular dependence of the bias is developed as a function of the normalized bias at nadir.

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# Chapter 1

# Introduction

In 1982 a weather phenomenon called *El Niño* caused intense changes in normal weather patterns across the globe. Off the South American coast water temperatures in the Pacific Ocean were close to  $10^{\circ}$  higher than normal, desert regions of Ecuador and Peru received over 100 inches of rain in six months, and the normally tranquil weather of Hawaii and Tahiti was interrupted by a number of typhoons. Across the Pacific the normal monsoon seasons in Asia and Australia were dry, resulting in drought conditions for most of the Far East.

These changes in weather patterns often come at a great cost in life and property. A particularly intense El Niño in 1877 killed over nine million people in China and another 8 million in India. In South America, heavy rains from the 1982 event caused mudslides that resulted in thousands of deaths, while the most recent event in 1997 caused property damage in the U.S. estimated at more than a billion dollars.

With increased knowledge and understanding of the geophysical processes that create global weather patterns, the effects of large scale disturbances can be mitigated. The *El Niño* in 1997 was the first time that scientists were able to predict such an event. With six months of warning, governments and people were able to prepare for the consequences and science was able to study the results of the weather disruptions.

Measuring and predicting long-term weather patterns is largely a result of the advent of satellite based remote sensing instruments. Altimeters, scatterometers, radiometers, and other instruments collect global measurements of the ocean topography, near surface winds, sea surface temperature, and other relevant data. As the accuracy of such measurements increase, our understanding of the forces of nature improves and our ability to accurately predict global weather patterns is enhanced.

Among the geophysical measurements used to create global weather models is the mean sea level (MSL). Conceptually, the methodology to remotely measure the mean sea level is simple. The basic operation of altimeters is to measure the time of flight of an electromagnetic pulse to reflect off the ocean surface and return. By converting the time of flight to a distance measurement the MSL can be calculated. Though simple in concept, the actual practice of measuring the MSL is complicated by variations in the satellite ephemeris, atmospheric interference, and interaction of the electromagnetic energy with the ocean surface.

With improved satellite technology the error budget of satellite measurements were reduced from the order of meters to tens of centimeters. Further refinements and a better understand of atmospheric and oceanic properties have reduced errors in MSL estimates to the centimeter order. In this dissertation I focus on the dominant error term remaining in mean sea level estimates: the electromagnetic (EM) bias.

#### 1.1 EM Bias Background

The EM bias is caused by a non-uniform return of electromagnetic energy over the height of ocean waves. For nadir looking instruments, the backscattered energy return from wave troughs is larger than that returned by the crests. This differential in energy is perceived by the altimeter as a delay in the pulse return, and the estimated surface height is biased low. A derivation showing the relationship of the bias definition to the working definition used in the tower experiments is shown in A.

### 1.1.1 Satellite-based Models

Empirical models developed from satellite-based altimeter measurements describe the EM bias in terms of remotely measurable ocean and environmental parameters. To first order these models all describe the bias as a linear function of the significant wave height, H, with higher order dependencies described as a function of other estimation parameters. Such definitions of the bias can be written as

$$\epsilon = Hf(\theta_p : U, H, ...), \tag{1.1}$$

where f describes the high order dependencies of the bias, and  $\theta_p$  represents the bias estimation parameters.

Because satellite-based empirical models are necessarily confined to using remotely measurable parameters, the estimation parameters must be derived from byproducts of the altimeter range estimates. The most common bias estimation parameters are wind speed, U, and significant wave height, though other parameters such as direct power measurements and a pseudo wave age have also been used.

The accuracy of bias estimates made using remote sensing instruments are limited by a lack of ground-truth bias measurements. Operational models for the satellite based altimeters, TOPEX/Poseidon and Jason-1, are derived from collinear or cross-track measurements of the surface [6, 7, 8, 9, 10]. After adjusting for known variations in the ocean height and atmospheric effects, such as tide, atmospheric pressure changes, propagation delays, and humidity changes, the difference in height between two measurements,  $\Delta r$ , is written as

$$\Delta r = r_1(U_1, H_1) - r_2(U_2, H_2) \tag{1.2}$$

$$= \epsilon(U_1, H_1) - \epsilon(U_2, H_2) + \delta, \qquad (1.3)$$

where  $\delta$  is assumed to be a zero mean noise process, uncorrelated to the EM bias. Using a least squares fit to the estimation parameters, an estimate of the bias is computed.

#### 1.1.2 Tower-based Models

EM bias models created from tower-based data have the advantage over satellite models of direct measurements of the EM bias, but are limited in spatial and temporal extent compared to satellites. Measurements of the bias are computed as the normalized correlation between the backscatter coefficient profile,  $\sigma^{\circ}(\zeta)$ , and the surface height,  $\zeta$ ,

$$\epsilon = \frac{E\left[\sigma^{\circ}(\zeta)\zeta\right]}{E\left[\sigma^{\circ}(\zeta)\right]},\tag{1.4}$$

where  $\sigma^{\circ}$  is a normalized backscatter measurement.

Due to the proximity of the tower experiments to the ocean surface, direct measurements of the surface profile,  $\zeta$ , and related backscatter measurements,  $\sigma^{\circ}$ , are available. Using these values in equation (1.4), the EM bias can be computed directly. The correlation of the EM bias values with simultaneous measurements of the related ocean and atmospheric estimation parameters, such as significant wave height and wind speed, is used to create the EM bias models.

Because tower-based EM bias models are created from ground truth data, they provide an important check to satellite-based models. As a check of satellitebased models, the first tower-based models described the bias as a function of the wind speed and significant wave height [11, 12]. More recent models have been developed by parameterizing the EM bias as a function of other ocean surface parameters [13].

### 1.1.3 Theoretical Modeling

Two physical effects have been shown to cause the EM bias: non-Gaussian surface statistics and hydrodynamic modulation. The first bias theories were developed using the weakly nonlinear theory of Longuet-Higgins [14] to describe the non-zero, higher order moments of the surface. These models described the EM bias as a function of the non-Gaussian height distribution, skewness, and the correlation between the height and tilt of the surface, tilt modulation [5, 15, 16].

More recently, other EM bias theories have described the bias as a function of small wave height modulation [13]. These models are implicitly based on the relationship of short wave modulation with the RMS slope described by Longuet-Higgins [17], and have been shown to greatly improve EM bias estimates.

#### **1.2** Dissertation Objectives

The objective of this dissertation is to develop methods to improve EM bias estimates for remote sensing instruments. Research topics include the analysis of current operational models, the development of a better bias theory, and the collection of *in situ* measurements. Measurements and theory are also expanded to off-nadir incidence angles, in anticipation of future satellite missions, such as the Wide-Swath Ocean Altimeter.

The first contribution of this dissertation results from an empirical analysis of EM bias modeling parameters. Because operational EM bias models are developed using empirical relationships, the accuracy of the model depends on the correlation of the bias with atmospheric and oceanic parameters. The analysis presented here provides two important results. The first shows that models developed from just the wind speed and significant wave height, are inherently noisy due to the weak correlation of these parameters with the relative bias. The second shows that the inclusion of the RMS wave slope as an estimation parameter can reduce error values to the sub-centimeter level. This study has been published in the *Journal of Geophysical Research* [18].

The next contribution investigates the improvement of EM bias models using nonparametric estimation. Typical bias models, developed as least-squares estimates over the entire parameter space, have the potential to be inaccurate in regions with small data densities. In contrast, nonparametric models are developed as a set of local estimates created from nearby data values, and can better estimate the data in low density regions of the parameter space. This modeling bias toward the regions of high data density causes satellite and tower models to differ on the order of 2-5 cm for average bias estimates. Using nonparametric modeling techniques, the historical differences between tower and satellite based models are significantly reduced, and similar models were created by both types of data. This study has been published in the *Journal of Geophysical Research* [19].

Another contribution in this dissertation is a validity study of rough surface scattering models, as these are used in theoretical EM bias models. Of primary importance is the region of validity of the physical optics scattering model for surfaces with a power law PSD. By numerically modeling the EM scattering from power law surfaces, the validity of the PO model is shown to be a function of the significant slope of the surface. Similar validity criteria are found for other models in the study, including geometrical optics. This portion of the dissertation research has been published in *Waves and Random Media* [20].

The development of an improved EM bias theory is the next contribution. Using the physical optics scattering model, the EM bias theory includes bias contributions from the two physical processes that have been shown to cause the bias: hydrodynamic modulation and the non-Gaussian long wave surface statistics. The final form of the model describes the relative bias as a function of long wave surface statistics with coefficients related to the short wave surface scattering. By including the PO scattering approximation in the derivation of the bias theory, the calculation of the models is greatly simplified relative to other theories. The final model is shown to be more accurate than previous bias theories, and provides an insight into the physical mechanisms that influence the EM bias. This research has been submitted to the *Journal of Geophysical Research* and is currently in review.

The next contribution is the deployment and analysis of the Brigham Young University Off-Nadir Experiment (Y-ONE). Measurements of bias at nadir and off-nadir incidence angles provide information on the EM bias. With the nadir measurements, the Y-ONE data is the latest in a set of tower-based EM bias experiments that have been used to corroborate theoretical EM bias models. Off-nadir measurements of the bias are made in anticipation of the launch of the Wide-Swath Ocean Altimeter. These measurements demonstrate the angular dependence of the bias such that increasing incidence angles result in a decrease in magnitude of EM bias measurements. Results from the Y-ONE experiment have been submitted to *Journal of Geophysical Research* and the article is currently in review.

The final contribution in the dissertation is the development of an off-nadir EM bias theory. The off-nadir theory is developed by generalizing the EM bias theory to off-nadir incidence angles. This theory develops an off-nadir approximation of the bias, and describes the angular dependence as a function of changes in the short wave scattering with incidence angle. A comparison of the theory with measurements from the EM bias experiments shows qualitative similarities. This research have been submitted with the Y-ONE experiment to *Journal of Geophysical Research* and is currently in review.

Each chapter in this dissertation develops one of the contributions to the EM bias modeling discussed above. Analysis of the EM bias modeling parameters is considered in Chapter 2. The development, application, and improvement of EM bias estimates with nonparametric modeling method is discussed in Chapter 3. Chapter 4 discusses the validity study of the rough surface scattering models, and the physical optics scattering approximation. In Chapter 5 the EM bias model is developed and analyzed, and the Y-ONE experiment and data are presented in Chapter 6. In Chapter 7 the nadir model is generalized to off-nadir incidence angles, and the angular dependence of the EM bias theory is described. Conclusions and future work are discussed in Chapter 8.

### Chapter 2

# **RMS Wave Slope**

Numerous studies on the EM bias have been conducted since its discovery by Yaplee, *et al.* [21]. Efforts to explain the theoretical mechanisms and physics that cause the EM bias have included laboratory experiments [22, 23], numerical analysis [24], and non-linear sea surface models [16, 25]. Theoretical models have been of limited usefulness due to the difficulties inherent in modeling nonlinear ocean surface hydrodynamics and improving on approximate electromagnetic scattering models such as physical and geometrical optics. Because of this, operational EM bias estimation relies on empirical models. Current empirical models are based on significant wave height and wind speed measurements from the TOPEX/Poseidon satellite mission [6, 7, 8, 9]. Other models created from satellite data have accounted for wave development by including a wave age or pseudo-wave age parameter to improve the bias estimates [26, 27]. Empirical models have also been obtained from aircraft [28] and tower experiments [11, 12].

The large amount of data collected from the TOPEX/Poseidon satellite have allowed very accurate models of the mean value of the EM bias to be created using crossover differences [9]. Mean-bias models are limited by large variability of EM bias as a function of significant wave height and wind speed. The ultimate goal of current research efforts is to reduce this variability by including additional measurable or predictable parameters in bias models.

The importance of hydrodynamic modulation in determining EM bias has been the subject of much study, especially in recent years [25, 27, 29, 30]. These theoretical studies indicate a relationship between EM bias and a higher order moment of the surface height power spectral density, such as RMS long wave slope, or a quantity that is closely related to a higher order moment, such as orbital velocity. In this chapter, I study the correlation between the RMS long wave slope and the EM bias. It is shown that the wave slope is more strongly correlated to the EM bias than either wind speed or significant wave height. Due to the stronger correlation between the EM bias and the wave slope, the errors between measured and estimated values are significantly reduced relative to traditional wind speed and wave height models. The wave slope is also shown to create accurate estimates of the EM bias over different sea states, wind speed ranges, and locations.

#### 2.1 Data Sets

One of the difficulties in creating EM bias measurements using remote sensing instruments is the lack of measured truth values. Previous EM bias studies have used a variety of methods to obtain accurate estimates. Gaspar, *et al.* [7] and Chelton [6] used cross-track or collinear estimation techniques, respectively, to create EM bias estimates from satellite data. Hevizi, *et al.* [28] used laser measurements in their airplane experiments to collect the data necessary for EM bias estimation.

The tower experiments of Arnold, *et al.* [12] in the Gulf of Mexico (GME) and Melville, *et al.* [13] in the Bass Strait, Australia (BSE) [31, 32] created direct measurements of the EM bias. Each experiment used a Ku-band altimeter to measure the apparent sea surface height. Concurrent measurements of the ocean surface were made using wave gauges. The EM bias was then calculated as the difference between the measured and apparent sea surface heights. Simultaneous measurements of the wind speed were also made.

In agreement with past experiments, the GME and BSE data sets show a roughly linear relationship between the significant wave height, H defined as four standard deviations of the surface height, and the EM bias, as shown in Figure 2.1. As is commonly done, the leading linear dependence on H is removed and I study the normalized EM bias

$$\beta = \frac{\epsilon}{H}.$$
(2.1)



Figure 2.1: Correlation between the EM bias and significant wave height for GME and BSE data sets. A strong linear correlation between the significant wave height and EM bias is evident.

Thus, the models studied in this chapter are of the form

$$\epsilon = a(U, H)H,\tag{2.2}$$

where a(U, H) is a normalized bias model and U is wind speed referred to a 10 meter height. Since values for the significant wave height typically range from 0.5 - 3 m for these data sets, a value of 1% of the significant wave height for the normalized bias ranges from 0.5 - 3 cm. After this point, the normalized EM bias is referred to as EM bias for brevity.

It will be seen that GME and BSE data sets show similarities that can be attributed to general traits in the oceans of the world as well as specific regional characteristics. The data sets are combined to form a third data set, referred to as TOT. The relationships between the models created from the three data sets are used to identify regionally applicable characteristics as well as the general features that are applicable to global EM bias estimates.

### 2.2 RMS Long Wave Slope and Hydrodynamic Modulation

Studies of the relationship between scattering from the ocean surface and hydrodynamic modulation have been an active area of research in recent years. A
number of results have provided strong empirical and theoretical support for the importance of hydrodynamic modulation as a determinant of the EM bias [25, 27, 29, 30]. Because of the difficulty of solving the nonlinear equations which govern sea-air interactions, a number of approaches to parameterizing hydrodynamic modulation have been suggested. Most treatments involve higher order moments of the surface height power spectral density. Surface slope variance has long been suggested as a candidate for including the effects of hydrodynamic modulations in EM bias models, on both empirical and theoretical grounds [27]. Later studies lend further support to the importance of surface slope [23]. Indeed, a simple computation based on a two-scale surface model and nonlinear hydrodynamic theory shows that to first order, the change in amplitude with displacement of small waves riding on long waves is in direct proportion to RMS long wave slope S [33]:

$$\frac{a(\eta)}{a(0)} \simeq 1 + \sqrt{2}S\eta/a_l \tag{2.3}$$

where  $\eta$  is surface displacement due to a long wave of amplitude  $a_l$ . These considerations motivate the use of RMS slope in this study.

Instead of slope variance or RMS surface slope, which can be obtained from the the second moment of the surface height PSD, EM bias dependence on the first moment of the surface height PSD, orbital velocity, has also been studied theoretically [25]. Orbital velocity can be related to the first moment of the surface height power spectral density (PSD). The dependence of EM bias on orbital velocity has also been studied using the tower data from the GME and BSE data sets. In fact, Figure 2.2 shows that the correlation between normalized EM bias and RMS wave slope is similar to that of orbital velocity. A more detailed comparison shows that models based on wave slope performed slightly better than models using orbital velocity. As the difference between both sets of models is slight, however, the results cannot be considered to support RMS slope over orbital velocity, but rather demonstrate that inclusion of some parameter related to a higher moment of the surface height PSD leads to models that perform significantly better than those based only on significant wave height and wind speed.



Figure 2.2: Normalized bias vs. RMS wave slope and normalized bias vs. orbital velocity. The RMS wave slope and orbital velocity are similar in their correlation with the normalized bias.

There is good reason for the similar correlation of EM bias with RMS slope and orbital velocity. To improve bias models depending only on significant wave height and wind speed, additional model parameters must include information not available from these two parameters. Significant wave height is dominated by the longest surface wave components at wavenumbers much lower than the EM wavenumber. Instantaneous wind speed determines the amplitude of capillary wave components with very high wavenumber, on the order of the EM wavenumber. Higher order moments of the surface height PSD tend to emphasize surface components near a cutoff frequency which is generally chosen to be less than the EM wavenumber and larger than the wavenumber of the longest waves. Thus, RMS slope and orbital velocity contain information about mid-range surface waves, smaller in wavelength than those represented in the significant wave height and longer in wavelength than Bragg-scale capillary waves driven by local winds.

Because surface height measurements in the BSE and GME experiments are performed as a function of time but at the same spatial point, the surface height wavenumber PSD is not directly available. Well-known and commonly used techniques allow the RMS surface slope to be obtained from the time frequency PSD,  $W(\omega)$ , instead. Cox and Munk [34] obtain the result

$$S = \left[\int k^2 W(\omega) d\omega\right]^{1/2} \tag{2.4}$$

where k is given by the surface wave dispersion relation

$$\omega^2 = gk + \gamma k^4. \tag{2.5}$$

Here,  $\gamma$  is the ratio of surface tension to density and g is the acceleration due to gravity. Since I am interested in medium to long wave surface components with wavelength on the order of one meter or longer, we neglect the second term of (2.5), since it is significant only for capillary waves of centimeter-scale and smaller. Discretizing the integral in (2.4) using an N-point Euler quadrature rule and making use of the dispersion relation leads to

$$S = \left[\Delta\omega\sum_{n=1}^{N_c} \frac{\omega_n^4}{g^2} W(\omega_n)\right]^{1/2}$$
(2.6)

where  $\Delta \omega = \omega_{max}/N_{FFT}$ ,  $\omega_{max}$  is the maximum time frequency at which the  $N_{FFT}$ PSD is computed, and  $N_c$  corresponds to the upper cutoff frequency,  $\omega_c$ . For the GME experiment, the sampling rate of surface height measurements is 8.2 Hz.

One of the chief difficulties in determining a meaningful RMS long wave slope is an appropriate choice of a cutoff frequency for the surface height displacement spectrum. For both the GME and BSE tower experiments, the radar footprint is small enough that each instantaneous backscatter measurement can be considered to be due to a single facet consisting of small, Bragg-scale waves tilted and modulated by longer waves. Thus, the long wave surface components can be considered to consist of wavelengths longer than the radar footprint. Using the gravity wave dispersion relation, this leads to a long wave cutoff frequency of roughly 1 Hz. Figure 2.3 shows that the typical time frequency surface spectrum deviates from the expected power law trend near 1 Hz. To avoid this region, the cutoff frequency for both experimental data sets is chosen to be  $f_c = 0.8$  Hz.

The mean RMS slope values for the GME and BSE experiments differ by a small factor, likely due to different choices for periodogram window lengths used to



Figure 2.3: Typical one hour averages of the surface height power spectrum, W(f), and slope spectrum S(f).

compute surface PSDs or other constants in the RMS slope calculation. Because the raw data for the BSE experiment is not available, the RMS slope values could not be recomputed using identical processing, so the GME slope values are multiplied by a factor of 0.67 to coregister the two data sets.

## 2.3 Qualitative Analysis

After the removal of the linear relationship between the EM bias and the significant wave height, it can be seen in Figure 2.4 that the normalized bias is only weakly correlated with the significant wave height in either experiment. The second-order polynomial, least squares estimates in the plots also show that the fit

to significant wave height is not consistent across the two experiments. The functional form and the zero significant wave height intercept of the estimates show large discrepancies.



Figure 2.4: Normalized bias,  $\beta$ , vs. significant wave height, H, showing a second-order least-squares fit to the data.

Figure 2.5 shows a more strongly correlated relationship between the wind speed, U, and the normalized bias. The plots show more regional correlation between the GME and BSE experiments, but as with the second order fit to H, the zero wind speed intercepts are significantly different for the two data sets.

Of the three parameters studied, the normalized EM bias is the most strongly correlated to RMS long wave slope, as seen in Figure 2.6. Although the improvement is less pronounced for the GME experiment, there is less scatter associated with the wave slope than with either wind speed or significant wave height for both data sets. The shape of the estimates is more consistent from one data set to the other, indicating that wave slope may explain regional variability in the EM bias. As with the significant wave height and wind speed second-order fits, the intercepts for fits to slope are also different for the two models. It is clear from inspection of the



Figure 2.5: Normalized bias,  $\beta$ , vs. wind speed, U, showing a second-order least-squares fit to the data.

data sets, however, that the trend for low slope values for both experiments is toward a very small or zero intercept value that is nearly identical in both cases.

### 2.4 Methodology

The general form of the EM bias models considered is

$$\beta = a_0 + \sum_{i=1}^{N} a_i P_i$$
 (2.7)

where  $\beta$  is the normalized bias, N is the number of terms in the model, and  $P_i$  is a linear or second-order term in the Taylor series expansion of a(U, H, S), so that  $P_i$  is chosen from the set  $U, H, S, U^2, H^2, S^2, UH, US$ , or HS. Writing the coefficients  $a_i$  in equation (2.7) in vector form  $\vec{a}$  and minimizing the RMS error of the model over a data set leads to the least squares solution

$$\vec{a} = (\vec{P}^T \vec{P})^{-1} \vec{P}^T \vec{\beta} \tag{2.8}$$

where  $\vec{P} = [1, P_1, \dots, P_N]^T$  and  $\vec{\beta}$  is a vector of measured normalized bias values.

By subtracting estimated values from measured values over a data set, a vector of error values is created. These are referred to as residual errors. The metric



Figure 2.6: Normalized bias,  $\beta$ , vs. wave slope, S, showing a second-order least-squares fit to the data.

used in determining the most effective models is the RMS value of the residual errors. This approach to determine the optimal combination of parameters is similar to that of Gaspar, *et al.* [7]. Models are created for each of the three data sets: GME, BSE, and TOT. These models are used to estimate each of the data sets. Residual errors are calculated and the models ranked according to the RMS residual error. Best-case models are also created for each of the data sets, with and without the wave slope parameter.

#### 2.5 Results

The effectiveness of the modeling parameters is evaluated by observing the contribution of the terms individually and in combination with others.

#### 2.5.1 One-Term Models

One-term models are defined to be of the form

$$\beta = a_0 + a_1 P_1$$

where  $P_1$  is one of the parameters  $U, H, S, U^2, H^2, S^2, UH, US$ , and HS.



Figure 2.7: One-term model performance. The wave slope, S, generally provides the best models, while the significant wave height, H, carries the least amount of information on the normalized EM bias.

The plots in Figure 2.7 show RMS error values for the differences between the measured normalized bias values and the one-term models indicated. Each of the plots shows the models for a given data set: GME, BSE, or TOT. The parameters on which the models are based are ordered for monotonically increasing RMS error values for the TOT data set.

The best one-term models of the normalized bias for each data set are created using the wave slope data. The BSE normalized bias values are seen to be especially well correlated with the S. Other terms which have contributions from the wave slope are also seen to be more highly correlated to the normalized bias. To calculate the normalized bias, the linear dependence on H is removed. The oneterm models show that there is much less bias information remaining in H than that contained in U and S. This is seen in Figure 2.7 by models containing contributions from H having the largest error values. Models containing wind speed information are more strongly correlated to the normalized bias than models based on H, but generally have larger RMS error values than models using S.

# 2.5.2 Two-Term Models



Figure 2.8: Two-term model performance. The best models show the complementary information that the wave slope, S, and significant wave height, H, parameters use to create more accurate models.

Two-term models provide significant improvement over the one-term models. For every combination of model and data set, the top two-term models significantly outperform the best one-term model. There is also a shift in the importance of the various parameters when two terms are used. The one-term models identified the



Figure 2.9: Best two-term models for each data set. When estimating the TOT data set, the terms in the best models are composed solely of the wave slope and significant wave height. Shaded areas indicate regions with no data.

wave slope and the wind speed as the parameters which contain the most information individually.

The two-term models identify the combination of parameters which contain complementary information. Figure 2.8 shows that the models with lowest error values are derived from terms containing S and H. Though U contains more information on the normalized bias than H, the two-term models indicate that it is redundant information to that contained in S. Conversely, the information contained in the H and S terms is complementary. Combinations of terms derived from S and H constitute the best five models in every case.

# 2.5.3 Three-Term Models

The 20 three-term models with the lowest RMS errors from each of data set is shown in Figure 2.10. The addition of a third term to the normalized bias models results in improvement over the two-term models on the order of 0.1% of the significant wave height. In addition, the differences in the RMS error values among the top three-term models is very small. Much of this is due to the most accurate models relying on the same terms. It can be seen that the wave slope S is included in the best 10 models regardless of the data set from which the models are created.



Figure 2.10: Best three-term model performance. The best models are predominantly derived from the wave slope, S, and significant wave height, H, parameters.

#### 2.5.4 Best Case Models

Models created with more than four terms result in very slight improvements in some cases, and in many cases the RMS error values increased. For all cases the RMS error values for the four-term models are within 0.012 %H of the best model for that combination of model and data set. The best four-term models are consistently those including the terms S, H,  $H^2$ , and HS.

For comparison purposes, the best models for each of the combinations of models and data sets are created without using the S parameter. The Taylor series expansion for these models yields five terms:  $U, H, U^2, H^2, UH$ . Figure 2.11 shows the results of these models and the best-case models using S.

The models including the wave slope parameter have RMS error values that are better by over 50% in every case but one, and every case shows improvements of at least 0.23 %H. For the more generally TOT model, the improvement is over 0.5 %H regardless of the data set to which it is applied.

### 2.6 Residual Errors

Figure 2.12 shows the residual errors for the best one-term, two-term, threeterm and best-case models for the TOT data set. The vertical line on the graph indicates the break between the GME and the BSE data. The x-axis is a chronological ordering of values, with each point representing a ten-minute average. Due to spurious data points in the two experiments there are jumps in the data sequence, so that the horizontal axis is a a pseudo-time representation of the data.

It can be seen from these figures that the residual plots show some remaining correlation in time. As the number of terms increases, not only do the errors decrease, but the correlation can be seen to decrease as well. However, even for the best-case model with slope, there are indications that the errors are correlated and show some time-dependent physical property of the wave field. Future research should investigate the cause of this correlation.



Figure 2.11: Residual errors for best-case models. The models derived using the wave slope show error values that are significantly smaller than those models that use only wind speed and significant wave height.

# 2.7 Conclusion

This study investigates the improvements in EM bias estimation using the RMS wave slope parameter. Measurements from the Gulf of Mexico Experiment (GME) and the Bass Strait, Australia Experiment (BSE) show regional differences in the wind speed and significant wave height parameters. The correlation of the EM bias with these parameters exhibits significant variability. Due to these characteristics, models based on the wind speed and significant wave height result in large residual errors when compared with truth data. Including RMS long wave slope as a model parameter reduces estimation error across a wide range of wind and sea conditions. The improvement in model correlation is evident over a variety of models based on wave

Model	Normalized Bias Residual Errors (%H)	RMS Error
TOT $(S)$	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} $	0.56% H
TOT $(S, H)$	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} $	0.39% H
TOT $(S, H, S^2)$	$\stackrel{\text{(Find)}}{\overset{(Find)}}{\overset{(Find)}}{\overset{(Find)}}{\overset{(Find)}}{\overset{(Find)}}{\overset{(Find)}{\overset{(Find)}}$	0.37% H
TOT $(S, H, S^2, SH)$	$\begin{array}{c c} & GME & BSE \\ \hline \\ & & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\$	0.33% H
TOT $(U, H, U^2, H^2, H)$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0.82%H

Figure 2.12: Residual error values for the best 1-4 term TOT models using wave slope and the best TOT model that does not use wave slope.

slope also reduce the regional variability of the models when the two experimental data sets from widely separated locations are combined. Slope-based models also improve cross-estimation of one data set using a model based on the other set. Typical improvements over wind speed and significant wave height-based models are on the order of 50% as measured by RMS error between predicted and truth EM bias values.

The information provided by the wave slope is also shown to be complementary to the significant wave height. Models based on combinations of the wave slope and significant wave height consistently result in the smallest error values between the measured and estimated bias values relative to models which included wind speed. RMS residual bias errors for these models are on the order of 0.34 %H.

As satellite altimetry instrumentation improves, the gains to be realized by improving EM bias estimates become increasingly important. Future satellite missions may have EM bias error budget contributions on the sub-centimeter level. Models based on the RMS long wave slope improve estimation errors to essentially this level. These results will obtain operational value if a means can be found to obtain remotely the wave slope or another sea-state parameter such as orbital velocity containing similar information on hydrodynamic modulation. Various means have been suggested for accomplishing this, including detailed studies of altimeter return signal waveforms and predictive wave models based on wind history, but conclusive results along these lines await further work.

# Chapter 3

# Nonparametric Estimation

### 3.1 Introduction

Due to the complexity and limited ranges of validity of theoretical EM models, empirical models are more commonly used in operational EM bias correction. These models are based on significant wave height and wind speed, as these parameters are available as a byproduct of altimeter measurements. With the launch of the TOPEX instrument, a number of models have been created from satellite data sets using crossover difference estimation techniques. The geophysical model function currently employed to estimate the EM bias for the TOPEX/Poseidon satellite mission includes the leading linear dependence of the significant wave height and an additional factor that is quadratic in wind speed. Other satellite-based models have included second-order terms in the significant wave height as well [6, 7]. Airplane and buoy data have also been used to develop models based on significant wave height, wind speed, and a derived value related to wave development [28].

Most existing empirical models are least squares fits of parametric models to wind and sea parameters. Due to the unequal distribution of data points collected in experimental studies, least squares models are biased in regions of low data density by parameter regions with large amounts of data. Nonparametric estimators have been suggested as a remedy [8]. The aim of this study is to evaluate the effectiveness of models based on nonparametric estimation techniques for tower and satellite-based data sets. These models are compared to parametric models to determine the degree of improvement of nonparametric estimation over traditional techniques.

The data sets employed in this study were obtained from tower experiments conducted by Arnold, *et al.* [12] in the Gulf of Mexico (GME) and Melville, et al. [13] in the Bass Strait, Australia (BSE) [31, 32]. Wave gauges and a Ku-band altimeter provided measurements of the sea surface height. By subtracting height measurements from the altimeter measurements, ground truth data of the EM bias was available for study. Simultaneous measurements of the wind speed were also collected.

#### 3.2 Nonparametric Regression

Commonly, the relationship between the EM bias and wind and ocean parameters has been estimated by a polynomial that minimizes the discrepancy between modeled and measured bias values. To leading order, the EM bias is linearly related to significant wave height. Theoretical studies have not yet yielded conclusively a form for the remaining dependence of EM bias on sea state. Because of this, there may be no strong motivation for the use of any particular polynomial form to relate normalized EM bias to sea state parameters. This motivates the technique of nonparametric estimation, which replaces a global polynomial estimate with local piecewise polynomial approximations. A smoothing function or kernel is used so that local polynomials do not depend on data outside their interval of support, and smoother models are obtained. Due to the increased number of degrees of freedom of nonparametric models, unexpected structure of data may be representable [35], without incurring the instability of high order global polynomial approximations, which exhibit phenomena such as ringing near sharp jumps or changes in slope.

Nonparametric estimators solve the conditional expectation

$$\hat{\beta}(\vec{x}) = E[\beta|\vec{x}] \tag{3.1}$$

where  $\vec{x}$  is the vector of known parameters and  $\hat{\beta}$  is the value to be estimated. The class of nonparametric estimators that I am interested in are local polynomial approximations of the function  $\beta$ . Nonparametric estimators of this type require a choice of kernel, kernel bandwidth, which determines the spread of the kernel, and degree of the local polynomial approximation.

#### 3.2.1 Kernel

Since most kernels used for nonparametric estimation have similar form, the choice of kernel tends not to have a significant impact on the effectiveness of the estimator. For this study the kernel used is Gaussian,

$$W(x) = C \exp\left[-\frac{1}{2}\left(\frac{x-x_0}{h_x}\right)^2\right],\tag{3.2}$$

where C is a normalizing constant, x is a given parameter,  $h_x$  is the kernel bandwidth for that parameter, and  $x_0$  is the point around which the local polynomial approximation is made.

#### 3.2.2 Kernel Bandwidth

In general, determination of an optimal kernel bandwidth is an open question in nonparametric estimation. Fan and Gijbels [36] obtain an optimal bandwidth,

$$h_x = \left[ C(K) \frac{\sigma^2(x)}{n f(x) r''(x)} \right]^{1/(2p+3)}, \qquad (3.3)$$

where  $\sigma^2(x)$  is the variance of the data, f(x) is the data density function, r''(x) is the second derivative of the regression function, n is the number of data points, p is the order of the local polynomial being used in the estimate, and

$$C(W) = \frac{\int\limits_{-\infty}^{+\infty} W^2(x) dx}{\left(\int\limits_{-\infty}^{+\infty} x^2 K(x) dx\right)^2}.$$
(3.4)

For a Gaussian kernel, C(K) is 0.281. The values for  $\sigma^2(x)$  and n are readily available and are shown in Table 3.1. It can be seen from equation (3.3) that the bandwidth varies locally according to the data density and the concavity of the regression function. The difficulty in using this equation for the bandwidth is in determining good values for the data density function and the second derivative of the regression function. Since the estimator used is a local estimator, the local data density is sufficient to estimate the density function. For the models in this work it is approximated by the number of measurements in the desired grid box normalized by the mean number of data points in each of the boxes. Values for r''(x) are dependent on the function being estimated, which is a priori unknown. To calculate r''(x) an iterative method is used. First, r''(x) is set equal to 1, then the local bandwidths are calculated and the model is created. From the model, the second derivative are computed by matching a second-order polynomial at each grid point. A new value for r''(x) is calculated, and the process is repeated. The large amount of scatter in the data for significant wave height values lower than 1 m, causes the bias estimates to vary widely, as seen in Figure 3.2



Figure 3.1: Iterative and approximate bandwidths computed as a function of significant wave height.

Due to the complexity of the iterative method for computing equation (3.3) another approach is investigated. It can be seen in Figure 3.3 that in areas where there is sufficient data, second order polynomial models show strong similarities to the nonparametric models in the given data sets. In place of the iterative approach a second order polynomial is matched to the entire data set. The second derivative of this function gives a constant r''(x), which is used in the calculation of the bandwidth. RMS error values from this method yield results at least as good as the results from the iterative method, and due to the simplicity of implementation, this is the method



Figure 3.2: Comparison of EM bias models using the iterative and approximate bandwidths.

used throughout. A comparison of the different bandwidth values can be seen in Figure 3.1.

Using multiple parameters increases the difficulty in calculating the optimal bandwidth. For multiple dimensions a Hessian matrix is required in place of r''(x). Fan and Gijbels [36] state that a unique solution for the optimal bandwidth is only available for positive definite Hessian matrices. The data from the two experiments cited does not meet this criterion. Instead a product of univariate kernels is used for the multivariate cases [36].

Table 3.1: Variances for Gulf of Mexico and Bass Strait, Australia data sets.

Data Sets	GME		BSE			ТОТ			
Data Points	1063			1539			2602		
Parameters	U	Н	S	U	Н	S	U	Н	S
Variance	5.7	0.25	$5.26^{*}$	11.1	0.30	8.22*	8.9	0.29	$7.60^{*}$

<sup>\*</sup> The variances for S are \*10<sup>4</sup>.

#### 3.2.3 Regression Estimator

The third component in a nonparametric estimator is the type of regression estimator used. For the case of local polynomial estimators, the degree of the local polynomial must be determined. Due to its simplicity of implementation and lack of bias due to variation in the data density, the estimator used in this study is a local linear regressor [36]. Local linear regression is a system of linear estimates of the desired data set for given grid points. Let K be the number of grid points at which we create the linear estimates, M be the number of parameters used in the estimate, and N be the number of data points. The local linear regression estimator is then

$$\underset{a^{k}(0)\dots a^{k}(M)}{\arg\min} \sum_{i=1}^{N} w_{n}^{k} \left[ \left( \beta_{n} - a^{k}(0) - \sum_{m=1}^{M} a^{k}(m)(x^{k}(m) - x_{n}(m)) \right) \right]^{2}$$
(3.5)

where  $x^k(m)$  is the value of parameter m at grid point k and  $x_n(m)$  are the N data points of parameter m,  $w_n^k$  is the kernel function,  $a^k(0)$  is a constant offset, and  $a^k(1) \dots a^k(M)$  are the slopes for each of the parameters at grid point k. If I let  $\beta$  be a vector of data measurements, the solution matrix is

$$\mathcal{A} = (\mathcal{X}^T \mathcal{W} \mathcal{X})^{-1} \mathcal{X}^T \mathcal{W} \beta \tag{3.6}$$

where

$$\mathcal{A} = \begin{bmatrix} a^{k}(0) \\ a^{k}(1) \\ \vdots \\ a^{k}(M) \end{bmatrix}, \ \mathcal{X} = \begin{bmatrix} 1 & y_{1}^{k}(1) & \dots & y_{1}^{k}(M) \\ 1 & y_{2}^{k}(1) & \dots & y_{2}^{k}(M) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & y_{N}^{k}(1) & \dots & y_{N}^{k}(M) \end{bmatrix},$$
(3.7)

$$\mathcal{W} = \operatorname{diag}\{w(y_n^k(m))\}\tag{3.8}$$

and

$$y_n^k(m) = x^k(m) - x_n(m).$$
 (3.9)

Additional information on applying nonparametric techniques to the EM bias problem can be found in [8] and [9].

#### 3.3 Results

Due to the strong linear correlation of the EM bias with the significant wave height, H, it is useful to define the non-dimensional value of normalized EM bias as

$$\beta = \frac{\epsilon}{H} \tag{3.10}$$

where  $\beta$  is expressed as a percentage of H. Throughout this section, the parameter being estimated is the normalized bias.

The results for the study are divided into sections for one, two, and three dimensional models. The first section uses each of the parameters wind speed, U, significant wave height, H, and RMS wave slope, S individually to estimate the EM bias. The second section discusses the three combinations where two parameters are used in the estimates, (U, H), (U, S), and (H, S). The last section shows the results of bias models based on all three parameters, (U, H, S). As a control group to the nonparametric models, a set of second-order least squares polynomial estimates are also shown. These models are collectively referred to as parametric models.

As stated earlier, the major emphasis of this study is to investigate the bias estimation improvements that can be gained by using nonparametric models. Comparison of the parametric and nonparametric models are made for the one, two, and three parameter cases. Graphic representations of the parametric and nonparametric models are shown and analyzed. Using the truth data provided by the tower experiments, a quantitative analysis is also presented.

The differences between the measured EM bias values and the estimated values from the models are referred to as residual errors. The RMS values for these residual errors are shown for the application of the various models to each of the data sets. Since the error is computed at the measurement points of the tower experiment data set, the RMS errors are non-uniformly weighted by the histogram of the data density of the various parameters.

A secondary emphasis of this study is to investigate the improvement that the wave slope parameter contributes to EM bias estimation [18]. The improvement



Figure 3.3: Parametric and nonparametric estimates of the normalized bias using significant wave height, wind speed, and wave slope. The shaded regions indicate areas of low data density (< 3 data points/division).

caused by the addition of the RMS slope parameter is included in the analysis of the models in each section.

In reference to the figures in this section, it should be pointed out that although one of the advantages of nonparametric estimation is improved modeling of low data density regions, if the data density is too low, results become spurious. To indicate regions of validity, model plots are shaded in areas where the data density is below a fixed threshold (< 3 data points/division).



Figure 3.4: Normalized bias RMS errors for the one parameter models. Each graph represents a different training set. The data set indicated near the top of the graph is the data against which the model is used. The parameters used are indicated at the bottom.

# 3.3.1 One Term

Figure 3.3 presents the parametric and nonparametric one-dimensional bias models. Qualitatively, the difference between the parametric and nonparametric models for the one-dimensional case is quite small. The nonparametric models show adjustments to the data which the parametric models do not exhibit, but the adjustments are slight. Not surprisingly, these adjustments have very little effect on the calculated RMS values seen in Figure 3.4.

When comparing the effectiveness of the parameters individually, it is clear that the bias model derived from wave slope values consistently provides better models than the wind speed and significant wave height. This result is expected since the relatively small amount of scatter seen in Figure 3.3 indicates a stronger correlation of the wave slope parameter and the EM bias. The similarities in the slope-based models from the different data sets also cause these models to be more generally applicable, leading to less error when cross-estimating.



Figure 3.5: Parametric and nonparametric GME models of the normalized bias using wind speed and the significant wave height. The units of the bias are percentage of the significant wave height.

# 3.3.2 Two Terms

Graphical representations of the two-dimensional models can be seen in Figure 3.5–3.10. The two dimensional cases show more clearly the differences between parametric and nonparametric estimates. The graphs show similarities, but the adjustments made by the local weighting functions are clearly evident.



Figure 3.6: Parametric and nonparametric BSE models of the normalized bias using wind speed and the significant wave height. The units of the bias are percentage of the significant wave height.



Figure 3.7: Parametric and nonparametric GME models of the normalized bias using wind speed and the RMS wave slope. The units of the bias are percentage of the significant wave height.



Figure 3.8: Parametric and nonparametric BSE models of the normalized bias using wind speed and the RMS wave slope. The units of the bias are percentage of the significant wave height.



Figure 3.9: Parametric and nonparametric GME models of the normalized bias using significant wave height and the RMS wave slope. The units of the bias are percentage of the significant wave height.



Figure 3.10: Parametric and nonparametric BSE models of the normalized bias using significant wave height and the RMS wave slope. The units of the bias are percentage of the significant wave height.

The results of the adjustments can be seen in the RMS error values for the different models, shown in Figure 3.11. The models are more closely tuned to the local conditions from which they are calculated. When the models are used to estimate the training set from which they came, e.g. the GME model estimating the GME data set, there is notable improvement. It is significant that the TOT two parameter models show improvement using the nonparametric model over all data sets and all parameters.

It is expected that the bias models from distinct regions should be similar once all relevant physical parameters are taken into account. A comparison of the bias models based on wind speed and significant wave height from the BSE and GME data sets, in Figure 3.5 and 3.6, shows significant differences. These differences are largely corrected in Figure 3.9 and 3.10, where the bias is estimated using the wave slope and significant wave height. The effect of these similarities is reflected in the RMS error values shown in Figure 3.11. The addition of the wave slope reduces the error values by over 50% in most cases. This improvement is evident when estimating



Figure 3.11: Normalized bias RMS errors for the two parameter models. Each graph represents a different training set. The data sets indicated near the top of the graph are the data against which the model is used. The parameters used are indicated at the bottom.

the original training set, as well as when models are used to cross-estimate other data sets.

# 3.3.3 Three Terms

The quantitative analysis in Figure 3.12 for three parameter models shows similar results as the one and two dimensional models. In most cases, the nonparametric models show some improvement. The effects of local wind and sea conditions are more evident in the three parameter models. The cross-estimates are all degraded by the local conditions of the training sets to which the nonparametric models are fit. However, the TOT models created from the combined data set show improvement over all cases. This emphasizes the improvements that can be made using nonparametric methods.



Figure 3.12: Normalized bias RMS errors for the three parameter models. The data sets indicated near the top of the graph are the data against which the model is used. The training data sets are indicated at the bottom.

Another attribute of the nonparametric methods that can be seen in the three parameter models is improvement using all available data. Millet, *et al.* [18] showed that the best parametric models are created using only the significant wave height and the RMS wave slope. Parametric models that include the wind speed along with the significant wave height and the wave slope cause the model performance to deteriorate. Figure 3.12 shows that nonparametric models are able to effectively apply new information included in the wind speed to improve the three dimensional models. Because of the greater number of degrees of freedom in the model, the nonparametric technique appears to better identify the unique information from each parameter that can improve the overall estimate.

#### 3.4 Satellite Modeling

Using satellite data to estimate the EM bias is somewhat more complicated than using tower data since altimeters are unable to measure the EM bias directly. A general method used to estimate the bias using satellite technology is described here. For a more detailed explanation see [8] and [9].

To create a bias model, sea surface height differences between the ascending and descending passes of a satellite are measured. The differences in the sea surface heights are modeled as changes in EM bias values plus other factors, which are assumed to be zero mean. In equation form this is

$$SSH_2 - SSH_1 = y = \phi(\vec{x_2}) - \phi(\vec{x_1}) + \epsilon$$
 (3.11)

where  $\vec{x}$  is a vector of sea-state related variables, in this case (U, H),  $SSH_i$  are sea surface height measurements at crossover points,  $\phi$  are the EM bias values, and the indices 1 and 2 refer to measurements made on the ascending and descending arcs, respectively. The conditional expectation of equation (3.11) can then be written as

$$E[y|\vec{x_2} = \vec{x}] = \phi(\vec{x}) - E[\phi(\vec{x_1})|\vec{x_2} = \vec{x}].$$
(3.12)

Similar to the NPR technique explained previously, a nonparametric estimator is chosen and equation (3.12) becomes a system of equations of the form

$$(I-A)\phi_1 = Ay. \tag{3.13}$$

This system of equations is singular, requiring a constraint value. This occurs because the estimate is obtained from crossover difference values, so that the absolute bias is not known. The method described by Gaspar, *et al.* [9] is to chose a data point of wind speed and wave height which is near the center of the data values and impose a reasonable value for the bias at that point. After imposing this constraint the system can solved.

As opposed to the limited data sets collected by the two tower experiments, there are vast amounts of data collected from TOPEX/Poseidon. This yields smoother models than those created from the tower experiments. It should be noted that the models created by Gaspar, *et al.* [9] are models of the EM bias, not the normalized bias, for convenience in allowing differences between satellite and tower models to be evaluated visually in units of lengths.



# 3.4.1 Satellite and Tower Models

Previous satellite and tower EM bias models have had unexplained discrepancies on the order of several centimeters. Figure 3.4 shows that using nonparametric methods have greatly reduced this discrepancy. The difference plots between the TOPEX/Poseidon satellite model and each of the tower data sets, shown in Figure 3.13, show that the latest nonparametric tower and satellite models have differences under 2.5 cm for all values and a large portion of the tower estimates are within 1.0 cm of the satellite model values. The similarity between the satellite and tower models allow the TOPEX/Poseidon model to be applied to the GME, BSE, and TOT data sets.



Figure 3.13: Difference plots between the satellite model and each of the data sets.

### 3.4.2 Residual Errors

Different than other remote sensing experiments, tower experiments provide a method to directly measure EM bias values. Figure 3.14 shows the differences between measured EM bias values and estimated model values, or residual errors. The x-axis is a chronological ordering of values, with each point representing a onehour average. Due to spurious data points in the two experiments there are jumps in the data sequence, so the horizontal axis is pseudo-time.

The application of nonparametric estimation techniques provide some improvement in the RMS error values. Figure 3.14 is a summary of the improvements from nonparametric modeling and use of the slope parameter. Even with these improvements there is still significant variance in the residual errors. Some of this error can be seen to be the result of regional differences in wind and wave conditions. Though the TOT model, created from the combined data sets, is generally effective, an offset can be seen for each set of residual values. The satellite is also relatively effective, but is limited in its effectiveness by other regional conditions not included in the specific tower data sets. These models are all limited by the variance of the wind speed and significant wave height in estimating the bias. The TOT nonparametric model using wave slope shows the largest improvement, in reducing the RMS error to under 50% of the error realized with the TOT model using wind speed and significant wave height.

#### 3.5 Conclusion

The results of this study indicate that nonparametric estimates of EM bias consistently provide more accurate models than similar parametric models, although in some cases the improvement is slight. Nonparametric models using only one parameter show little improvement. Two and three parameter models show that nonparametric techniques reduced the normalized bias RMS error by up to 0.03% and 0.07% of the significant wave height, respectively.

The improvement of EM bias models with the inclusion of the RMS long wave slope parameter is also demonstrated. For the two parameter least squares TOT



Figure 3.14: RMS error bounds for the EM bias of satellite data applied to each of the data sets.

model, the best normalized bias RMS error using the wind speed and significant wave height is 0.82%. The RMS error for a similar two parameter using the wave slope and significant wave height is reduced to 0.34%. The addition of a third parameter, the wind speed, in the parametric models created negligible improvements. Nonparametric modeling extracts more bias information to reduce the normalized bias RMS error to 0.31% of the significant wave height.

Though models using the wave slope parameter are greatly improved over previous models, there still remains some variance when applied to the data. The variance can be accounted for by a correlation in time of the residual errors. This correlation indicates the possible existence of further sea state parameters that contain information on the EM bias. The models presented in this study also reduce the historical differences between satellite and tower EM bias estimates. The nonparametric estimation techniques support the reliability of both satellite and tower data sets in estimating the EM bias. Due to this correlation, the *in situ* data is able to provide quality assurance values for the satellite estimates. Applying the satellite model to the TOT data set gives an EM bias RMS error value of 1.41 cm. This is larger than the 0.80 cm error for the combined tower data model, but much smaller than the 5 – 6 cm differences in past studies. In areas of sufficient data density the differences between the TOT and satellite models are under 2 cm, and usually less than 1.5 cm.
# Chapter 4

# **Rough Surface Scattering**

### 4.1 Introduction

Modeling the interaction between electromagnetic waves and random rough surfaces is a problem of great importance in many applications. Due to the difficulty of obtaining exact scattering solutions, simplifying approximations are made to model the fields scattered by rough surfaces. Application of approximate scattering models requires an understanding of the regime of accuracy of the model in terms of the electromagnetic frequency, incidence angle, and statistical properties of the rough surface. Although a number of theoretical and empirical studies have been performed, the ranges of validity of many existing models have not been fully characterized, and the determination of validity criteria for rough surface scattering models continues to be an active area of research.

Early rough surface scattering models include the physical optics (PO) approximation, which is valid in the limit of large electromagnetic frequency, and the small perturbation method (SPM) in the low frequency limit. The geometrical optics model (GO) can be obtained as the infinite frequency limit of the PO model. These classical models continue to be used in many applications due to their robustness and simplicity of use.

More recently, as the importance and variety of applications of rough surface scattering have grown, new scattering models have been developed. Many of these models are based on perturbation expansions, and are typically named for the small parameter about which field quantities are expanded. These include higher order small perturbation methods, small slope approximation (SSA) [37, 38], phase perturbation [39], momentum transfer expansion [40], operator expansion [41, 42], and the unified perturbation expansion [43]. Iterated physical optics (ITPO) or the second-order Kirchhoff approximation [44] consists of the first two terms of a Neumann expansion of the magnetic field integral equation.

Because many of these scattering models reduce to PO at the lowest order, a thorough understanding of the PO approximation can provide insight into the accuracy of more sophisticated models. For this reason, I focus particular attention in this study on the PO model. The validity criterion [45],

$$kr_c \cos^3 \theta \gg 1,\tag{4.1}$$

where  $r_c$  is the radius of curvature, k is the electromagnetic wavenumber and  $\theta$  is the incidence angle, can be derived from the geometry of the interaction between the incident wave and the scattering surface. Ulaby *et al.* [1] refine the criterion to

$$kl > 6 \tag{4.2}$$

$$kr_c > 2.5,$$
 (4.3)

where l is the surface correlation length, for bistatic scattering.

Numerical studies have been conducted to further refine the limits of validity of the PO model. A study by Thorsos [2] of surfaces with a Gaussian power spectral density (PSD) supports the theoretical validity condition, kl > 6, for surfaces with small slope values and incidence angles away from grazing. Later results by Thorsos and Jackson [46] give

$$kl > 6 \tag{4.4}$$

$$kh \le 1,\tag{4.5}$$

where  $h^2$  is the surface height variance, as bounds on the validity of the PO model for bistatic scattering, but note that near the specular direction the model remains accurate for much smaller kl values. This increased accuracy for near-nadir incidence angles has been observed by Chen and Fung [3].

Significantly fewer studies have been conducted for surfaces with a power law PSD. For such surfaces Rodriguez *et al.* [27] show that for  $kh \leq 1$  the PO model is accurate at incidence angles less than 15°. Theoretical studies have shown that the PO model is accurate for backscattering at nadir incidence from power law surfaces with significant slope less than 0.03 [47, 48]. The numerical experiments reported in this study corroborate this result. Further discussion of previous work on PO validity criteria can be found in Section 4.5 of this chapter.

Studies have also provided validity criteria for the SPM, SSA, and ITPO models. From the derivation of SPM, the validity criterion  $kh \ll 1$  is obtained. Other refinements on the basic validity criterion include limits on kl [46, 3, 49] and variation with incidence angle [27]. Studies of the SSA model have shown that for Gaussian surfaces the fourth-order SSA model is accurate for surfaces with RMS slope values less than 0.5, and that for power law surfaces the higher-order SSA models are consistently more accurate than PO [38, 50]. The ITPO model has been shown to be accurate over a large range of surface profiles, with limitations at small grazing angles [46].

The primary result of this chapter is a more precise determination of the accuracy and ranges of validity of the PO, SSA, ITPO, and SPM models for backscattered fields. An exact formulation of electromagnetic scattering is used to compute backscatter values for a highly populated parameter space of surfaces with Gaussian and power law PSDs. Differences between the reference solution and the model values are used to define the accuracy of the models and to determine empirical validity criteria for the two surface types.

Using this approach the ranges of accuracy of the PO, SPM, GO, ITPO, and SSA models are determined for surfaces with Gaussian and power law power spectra. For Gaussian surfaces the range of validity of the PO model is shown to be a function of the RMS surface slope. Similar behavior is found for the GO, ITPO, and SSA models. The validity criterion for power law surfaces is shown to be a function of the significant slope (RMS surface height divided by the wavelength of the spectral peak of the PSD). The PO validity criteria for both surface types are also shown to have a  $\cos^3 \theta$  angular dependence. The study begins by defining the scattering problem and providing a statistical description of the surfaces. This is followed by a brief review of the derivation of the models in the study. The next section includes the technical details of surface generation and the method of moments reference solution. Following is an investigation of the region of validity of the PO model for Gaussian and power law surfaces. Included in this section is a discussion of the accuracy of the curvature validity criterion and the formulation of improved validity criteria based on the RMS surface slope and the significant slope. The next section discusses the accuracy, angular dependence, and validity criteria of the SPM, GO, ITPO, and SSA scattering models. The last section summarizes the results of the numerical study and draws conclusions about the validity criteria for the models in this study.

#### 4.2 Rough Surface Scattering

I investigate the scattering of a horizontally polarized, or transverse magnetic, plane wave from a perfectly conducting one-dimensional surface into twodimensional space, Figure 4.2. The incident wave is time harmonic and traveling in the x-z plane. The electromagnetic wavenumber is defined as  $k = 2\pi/\lambda$  and the incident electric field as

$$\vec{E}^i = -\hat{y}E_o e^{ik_x x - ik_z z},\tag{4.6}$$

where  $\theta$  is the incidence angle,  $k_x = k \sin \theta$  is the *x*-component of the incident wavevector, and

$$k_z = \sqrt{k^2 - k_x^2}.$$
 (4.7)

The convention used throughout this study is that k refers to the electromagnetic wavenumber and K refers to the surface wavenumber. All surfaces in this study have a Gaussian height distribution, and I consider two surface height power spectral densities: Gaussian and power law.



Figure 4.1: Scattering geometry for this study.

#### 4.2.1 Gaussian Surfaces

Gaussian surfaces are defined statistically by the correlation length, l, and the surface height variance,  $h^2$ . The surface height PSD is described by

$$W(K) = \frac{lh^2}{(2\sqrt{\pi})} e^{(-K^2 l^2/4)}.$$
(4.8)

Because the PSD is band-limited to lower wavenumbers, these surfaces are singlescale. The parameter space for the Gaussian surfaces is described by the dimensionless quantities kh and kl.

#### 4.2.2 Power Law Surfaces

Surfaces described by a power law PSD,

$$W(K) = \begin{cases} h^2(p-1)K_{min}^{(p-1)}K^{-p} & K \ge K_{min} \\ 0 & K < K_{min}, \end{cases}$$
(4.9)

are characterized by the surface height variance,  $h^2$ , the lower cutoff wavenumber,  $K_{min}$ , and the exponent of the spectrum, p. This type of surface is multi-scale and is representative of many surfaces found in nature, such as ocean surfaces.

The multi-scale nature of power law surfaces is due to the slowly decaying tail of the PSD. As a result of this slow decay, surfaces with an exponent  $p \leq 3$  have a mathematically infinite slope variance. The exponent p = 3 has been chosen as a typical value for ocean surfaces [51, 52], and discuss the implications of the infinite slope variance in a later section. For a fixed exponent, the power law parameter space is defined by the dimensionless variables kh and kL, where  $L = 2\pi/K_{min}$  is the dominant wavelength.

#### 4.2.3 Deterministic Scattering Coefficient

Deterministic scattering methods return the complete, coherent and incoherent, scattering field for each surface instantiation. For a single surface the 2D scattering coefficient can be computed from the scattered fields as

$$\sigma^{\circ} = \lim_{\rho \to \infty} \frac{2\pi\rho}{L} \left| \frac{E_z^s}{E_o} \right|^2.$$
(4.10)

To compute the average incoherent 2D scattering coefficient, the coherent scattering is subtracted from the total power, so that

$$\sigma_{inc}^{\circ} = \lim_{\rho \to \infty} \frac{2\pi\rho}{L} \left( \left\langle |(E_z^s)|^2 \right\rangle - |\langle E_z^s \rangle|^2 \right)$$
(4.11)

$$= \frac{k\eta_0^2}{4L} \left( \left| \left\langle (E_z^s)^2 \right\rangle \right| - \left| \left\langle E_z^s \right\rangle^2 \right| \right)$$
(4.12)

where  $\langle \rangle$  indicates the ensemble average.

#### 4.3 Models

The scattering models in this study are divided into two general types. Stochastic models use formulae derived from the statistical properties of the surface to calculate the average backscattered power from the surface. Deterministic models compute backscatter values for a single surface and are averaged, using the Monte Carlo approach, over many realizations to approximate stochastic results. One of the limiting factors of deterministic models is a much larger computational cost than stochastic models.

The stochastic models in this study are physical optics (PO), geometrical optics (GO), the small perturbation method (SPM), and the fourth order small slope approximation (SSA). The deterministic methods are Monte Carlo physical optics (MCPO) and iterated physical optics (ITPO).

## 4.3.1 Physical Optics

Physical optics (PO) is the oldest and most well-known rough surface scattering model. It is based on the premise that the surface can be modeled as a series of tangent planes which approximate the actual surface. Using the tangent plane approximation, so that  $\vec{H}^s = \vec{H}^i$ , the scattered electric field can be written as

$$E_{z}^{s} = \frac{k\eta_{0}}{4} \int_{S} J_{z}^{s}(\rho')g(\rho,\rho')\mathrm{d}S'$$
(4.13)

where

$$\vec{J^s} = \hat{n} \times (\vec{H^i} + \vec{H^s})$$
$$\approx 2\hat{n} \times \vec{H^i}, \qquad (4.14)$$

 $g(\rho, \rho')$  is the Green's function for the appropriate geometry and  $J_z^s(\rho')$  is the surface current.

The Monte Carlo physical optics (MCPO) model computes the scattered electric field by application of equation (4.13) and equation (4.14) to a deterministic surface. Averaging over many surface realizations leads to statistical moments of the scattered field.

For the stochastic PO model, the 2D scattering coefficient can be calculated from

$$\sigma^{\circ} = \lim_{\rho \to \infty} \frac{2\pi\rho}{L} \left\langle \frac{E_z^s E_z^{s*}}{E_0^2} \right\rangle$$
$$= \frac{k\cos^2\theta}{L} \iint dx dx' e^{ik_b x} \left\langle e^{i2k_z[\eta(x) - \eta(x')]} \right\rangle$$
(4.15)

where  $k_b = 2k \sin \theta$  is the Bragg wavenumber and  $\eta(x)$  is the surface profile. Further simplifications and algebra lead to [53]

$$\sigma^{\circ} = k \cos^2 \theta \int_{-L}^{L} \mathrm{d}x \left(1 - \frac{|x|}{L}\right) \mathrm{e}^{ik_b x} \mathrm{e}^{-\chi^2 [1 - C(x)]} \tag{4.16}$$

where C(x) is the correlation coefficient,  $\chi = 2hk_z$ ,  $h^2$  is the surface height variance, and the scattering coefficient is expressed in terms of the known stochastic properties of the surface, C(x) and  $h^2$ . It should be noted that equation (4.16) is the total backscatter coefficient of the surface. For incoherent backscatter equation (4.16) is modified to be

$$\sigma_{inc}^{\circ} = k \cos^2 \theta \int_{-L}^{L} dx \left( 1 - \frac{|x|}{L} \right) e^{ik_b x} \left( e^{-\chi^2 [1 - C(x)]} - e^{-\chi^2} \right)$$
(4.17)

where  $e^{-\chi^2}$ , which represents the coherent scattering, is subtracted from the kernel.

#### 4.3.2 Geometrical Optics

Geometrical optics (GO) is the infinite frequency limit of the PO approximation. If the slope variance of the surface is finite the correlation function C(x) can be approximated as  $1 - C''(0)x^2/2$ , and equation (4.16) reduces to

$$\sigma^{\circ} = k \cos^2 \theta \exp\left[\frac{-k_b^2}{2\chi^2 |C''(0)|}\right]$$
(4.18)

where |C''(0)| is the normalized slope variance. For power law surfaces with  $p \leq 3$ ,  $|C''(0)| = \infty$  if there is not a finite upper frequency cutoff. I have chosen  $K_{max} = 7k$  for the cutoff wavenumber of the power law surfaces.

### 4.3.3 Iterated Physical Optics

Iterated physical optics (ITPO), also known as the second-order Kirchhoff approximation, is based on the magnetic field integral equation

$$\hat{n} \times \vec{H}^{i}(\rho) = \left(2I - \nabla \times \int g(\rho, \rho')\right) \vec{J}^{s}(\rho')$$
  
=  $(I - M)\vec{J}^{s}$  (4.19)

where M denotes the integral operator in equation (4.19). The ITPO model uses a power series expansion of  $(I - M)^{-1}$ ,

$$(I - M)^{-1} = I + M + M^2 + M^3 + \dots$$
(4.20)

Assuming that M is small in some sense, the first two terms are sufficient to approximate the series and the surface current becomes

$$\vec{J}^s \approx (I+M)(2\hat{n} \times \vec{H}^i). \tag{4.21}$$

From the surface current the scattered fields can be computed using equation (4.13). I note that this model has been shown to have a non-physical growth in the normalized backscattering coefficient with increasing surface length [46].

## 4.3.4 Small Perturbation Method

While PO is a high frequency asymptotic approximation, the small perturbation method (SPM) is the classical low frequency approximation. The small perturbation method can be derived from equation (4.13) where  $g(\rho, \rho')$  is expressed as the Weyl plane-wave expansion of the Green's function and the surface current is written as

$$J^s = C \mathrm{e}^{ik_x x} f(x). \tag{4.22}$$

From this expression, f(x) and  $e^{ik_xx}$  are expanded in a power series, like terms are collected, and the scattered field is calculated. The zeroth order term,

$$\sigma^{\circ} = \frac{4k_z^4}{k}W(2k_x),\tag{4.23}$$

is the classic formulation of the SPM model.

## 4.3.5 Small Slope Approximation

The small slope approximation describes the scattered electric field as

$$E^s = \int \mathrm{d}k_x \exp^{i(k_x x + k_z z)} T(k_x), \qquad (4.24)$$

where the T matrix is

$$T = -\frac{1}{2\pi} \int dx \Phi e^{i2(k_x x + k_z z)} \bigg|_{z=\eta(x)}.$$
 (4.25)

The perturbation term,  $\Phi$ , is expanded in a power series about a generalized surface slope, and terms of a similar order of the slope term are grouped. The inclusion of each successive order of slope provides a more accurate result. The first expansion term of the SSA model, which is second-order in slope, is identical to the PO model in equation (4.16). The formulation of the fourth order approximation of the SSA model is

$$\sigma_4^{\circ} = \sigma_{00} + \sigma_{01} + \sigma_{11} \tag{4.26}$$

where  $\sigma_{00}$  is the PO model in equation (4.16) and  $\sigma_{01}$  and  $\sigma_{11}$  are defined by

$$\sigma_{01} = \frac{k_z^3}{\pi k} e^{-\chi^2} \operatorname{Re} \left\{ -J^* \int dx \ e^{ik_b x} B(x) + \int dx \ I(x) \int dK \ e^{iKx} W(K) g^*(K) \right\}$$
(4.27)  
$$\sigma_{11} = \frac{k_z^4}{2\pi k} e^{-\chi^2} \left\{ |J|^2 \int dx \ e^{ik_b x} B(x) + \int dx \ I(x) \int dK \ e^{iKx} W(K) + \int dx \ I(x) \int dK \ e^{iKx} W(K) + \int dK' \ e^{iK'x} W(K') g^*(K') \right\} \right\}$$
(4.28)

with

$$I(x) = e^{ik_b x} e^{\chi^2 C(x)}$$
  

$$B(x) = e^{\chi^2 C(x)} - 1$$
  

$$J = \int dK W(K) g(K)$$
  

$$g(K) = k \left( \sqrt{k^2 - (K - k_x)^2} + \sqrt{k^2 - (k_x - K)^2} \right) - 2k_z.$$

Due to the complexity of the model, higher order SSA approximations require more computational time than the other stochastic models. The third order small slope approximation is not included in this study, but full details can be found in [37] and [38].

#### 4.4 Numerical Implementation

The models studied are compared against an exact method of moments (MOM) solution of the scattering problem [54]. Here I describe the implementation details used to generate the reference scattering solutions.

## 4.4.1 Surface Generation

The random rough surfaces are generated from Gaussian and power law PSDs, using a spectral method described by Fung and Chen [55]. Surfaces are created

with surface length  $L_s \approx 80 \lambda_{em}$ , and sample distance  $dx = \lambda_{em}/10$ , with an incident wavenumber k=100 rad/m.

For Gaussian surfaces, h and l range from 0.01-0.2 m and 0.01-0.1 m, respectively, with increments of dh=0.01 m and dl=1/300 m. Power law surfaces have the same range of surface heights and L ranges from 0.1-3 m with increments of dL=0.1 m.

The Gaussian and power law parameter spaces are presented with kh values on the horizontal-axis and kl or kL on the vertical-axis. In this format the smoothest surfaces occupy the upper left corner of the parameter space. Conversely, the roughest surfaces have large kh values and small correlation lengths and are found in the lower right corner.

## 4.4.2 Method of Moments Reference Solution

The method of moments formalism for scattering from a rough surface for a TM wave provides an approximate solution to the electric field integral equation

$$E_z^i = \frac{k\eta_0}{4} \int_S J_z^s(\rho') H_0^{(2)}(k|\rho - \rho'|) \mathrm{d}S'.$$
(4.29)

Discretizing this equation leads to

$$\begin{pmatrix} E^{i}(\rho_{1}) \\ E^{i}(\rho_{2}) \\ \vdots \\ E^{i}(\rho_{N}) \end{pmatrix} =$$

$$\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{pmatrix} \begin{pmatrix} J_{z}(\rho_{1}) \\ J_{z}(\rho_{2}) \\ \vdots \\ J_{z}(\rho_{N}) \end{pmatrix}$$

$$(4.31)$$

where

$$A_{mn} \approx \begin{cases} \frac{k\eta_0}{4} \Delta S H_0^{(2)}(k|p_n - p_m|) & m \neq n \\ \frac{k\eta_0}{4} \Delta S \left[1 - j\frac{2}{\pi} \ln\left(\frac{\gamma k \Delta S}{4e}\right)\right] & m = n \end{cases},$$
(4.32)

 $\gamma \simeq 1.781$ , and  $\eta$  is the intrinsic impedance of free space. The surface current,  $J_z$ , is then computed from  $A^{-1}E$ . Using equation (4.13) the scattered fields can be calculated from the results for  $J_z$ .

#### 4.4.3 Electric Field Tapering

To eliminate edge effects that accompany finite length surfaces, a Gaussian windowing function, G(x), has been applied to the incident electric field. Acceptable angular resolution and stability can be obtained with the taper function

$$G(x) = \exp\left[-\left(\frac{x - L_s/2}{g}\right)^2\right]$$
(4.33)

where  $g = L_s / 12$  [56].

#### 4.4.4 Monte Carlo Convergence Study

The use of Monte Carlo averaging to approximate stochastic quantities introduces an inherent variance in the results. Because the reference solution is deterministic, the accuracy of the error values will be limited by the variance of the Monte Carlo average. Figure 4.2 shows some typical PO and MOM backscatter values for power law surfaces with kL = 150 and several kh values. The variation in the MOM solutions, as compared with the stochastic PO model, is evident.

To determine the convergence rate of the reference solution, 7500 realizations of a power law surface with kh = 10 and kL = 150 are created to calculate backscatter values. The results, seen in Figure 4.3, show that Monte Carlo errors decrease as the square-root of the number of realizations. As a trade-off between computational time and accuracy 50 surfaces are used for the Monte Carlo simulations, corresponding to an RMS error of 0.35 dB. The convergence rates for surfaces with different kh and kL values give similar results.

#### 4.4.5 Incidence Angles

Backscatter values for the reference solution and the models are computed at angles between 0° and 60° with an angular increment of  $d\theta = 0.25^{\circ}$ . I define



Figure 4.2: Typical PO and MOM backscatter values for power law surfaces with kL = 150 showing the variation inherent in the Monte Carlo averaging. The MOM reference solution is shown in black with the PO model shown in gray.

 $\sigma_{MODEL}^{\circ}$  and  $\sigma_{MOM}^{\circ}$  as the backscatter values in dB for the model and reference solution, respectively, and compute error values as

$$E(\theta_0) = \left(\frac{1}{5} \sum_{n=-2}^{n=2} \left[\sigma_{MODEL}^{\circ}(\theta_0 + n\mathrm{d}\theta) - \sigma_{MOM}^{\circ}(\theta_0 + n\mathrm{d}\theta)\right]^2\right)^{1/2}, \qquad (4.34)$$

where error values are averaged over 1° increments to reduce noise in the reported values.

# 4.5 Physical Optics

Since the development of the PO model, a number of different validity criteria have been presented. Methods to determine the region of validity of the PO model have included theoretical studies, empirical and numerical results, and physical



Figure 4.3: The RMS error values for successive numbers of surface instantiations with Monte Carlo averaging of the scattering coefficient. In this study N = 50 surfaces have been used for an error of 0.35 dB

reasoning. I give special attention to the PO model because many recent models have been shown to reduce to PO at lowest order.

In this section I review various PO validity criteria, and compare numerical results to the classic curvature validity criterion. I develop a validity criterion based on the RMS slope for surfaces with a Gaussian PSD, and present a similar validity criterion for the surfaces with a power law PSD based on the significant slope. Both validity criteria include a term related to the angular dependence.

## 4.5.1 Previous Studies

Theoretical studies of the limits of validity of the PO model have resulted in many different validity criteria. The most commonly used criterion for the accuracy of the PO model is equation (4.1). This result can be derived from the geometry of the rough surface scattering problem. Another criterion that has been derived from the geometry [57],

$$r_c \cos \theta \gg \lambda_{em},$$
 (4.35)

shows a similar dependence on the surface curvature, but has a different angular dependence. From integral equation techniques Meecham [58] requires that the maximum value of the surface slope be less than unity and that  $kr_c \gg 1$ . For a sinusoidal surface McCammon and McDaniel [59] limit the accuracy of the model to surfaces with small heights, long correlation lengths, low frequencies and incidence angles less than 60°, while Ogilvy states that the ratio of the electromagnetic wavenumber to the largest surface wavenumber is the most important parameter in determining the accuracy of the PO model [60].

Other studies have used computer simulations to numerically model the wave-surface interaction of rough surfaces. Commonly, surfaces used in such studies have a Gaussian PSD. As noted in the introduction, limits on the validity of the PO model have been proposed in many studies [2, 3, 38, 46]. Validity criteria and specific data points from a number of studies are reproduced in Figure 4.4. I note that the lower bound on kl obtained by several researchers is required for accuracy over a large range of bistatic scattering angles. As indicated by the results of Chen and Fung [3], PO can be accurate for values of kl below the limits observed in the bistatic studies if attention is restricted to near-nadir incidence angles.

### 4.5.2 Curvature Criterion - Gaussian Surfaces

The validity criterion, equation (4.1), for rough surface scattering is easily applied to surfaces with a Gaussian power spectrum. Because such surfaces are bandlimited, the integrated curvature of a surface, defined as the fourth moment of the power spectrum [1], can be computed analytically as

$$c = \sqrt{\int K^4 W(K) \mathrm{d}K} \tag{4.36}$$

$$= 2\sqrt{3}\frac{h}{l^2}, (4.37)$$



Figure 4.4: Numerical results of PO validity studies for bistatic scattering from surfaces with a Gaussian PSD. The region of validity for each of the studies is to the left and above the lines shown for Ulaby [1], Thorsos [2], and Chen and Fung [3]. Data points are reproduced from the study by Chen and Fung [3] for near-nadir incidence angles, where each  $\mathbf{o}$  indicates a valid data point, and + indicates an invalid data point.

where W(K) is the surface height power spectrum. From equation (4.1) and the definition of the integrated radius of curvature as  $r_c = 1/c$ , the curvature criterion for Gaussian surfaces becomes

$$\frac{kl^2}{2\sqrt{3}h}\cos^3\theta \gg 1. \tag{4.38}$$

To apply this criterion to numerical results equation (4.38) can be rewritten as

$$\frac{kl^2}{2\sqrt{3}h}\cos^3\theta > A_c^2$$

$$kl > A_c \left(\frac{2\sqrt{3}}{\cos^3\theta}\right)^{\frac{1}{2}}\sqrt{kh}$$
(4.39)

where the validity condition, expressed as a function of the surface parameters kh and kl, describes a curve in a two dimensional parameter space. The electromagnetic curvature,  $kr_c$ , is shown in Figure 4.5, along with lines of constant curvature for  $A_c = 1, 2$ , and 3.

#### 4.5.3 Numerical Results - Gaussian Surfaces

To evaluate the accuracy of the curvature validity criterion, the curve described in equation (4.39) is compared with the results of the numerical study. The plots in Figure 4.6 show the error values of the PO model with respect to the reference in gray-scale at incidence angles of  $0^{\circ}$ ,  $5^{\circ}$ ,  $20^{\circ}$  and  $45^{\circ}$ . A lower cutoff of 2 dB is imposed as the limit on the accuracy of the PO model.

The comparison of the error values with the curvature validity criterion, shows a qualitative difference between the numerical error values and the limit described by the classic curvature criterion, equation (4.1). At all incidence angles, the range of validity of the PO model is more accurately modeled by a straight line in the (kh,kl) parameter space. Included in each of the plots in Figure 4.6 is a solid line that corresponds to the curvature criterion in equation (4.39), where  $A_c = 2.2$  is the empirical best fit to an error threshold of 2 dB.

Other observations that can be drawn from Figure 4.6 include the validity of the PO model restricted to the region of the parameter space with smoother surfaces. Additionally, the error values can be seen to increase quickly through a fairly narrow transition region, after which the error values reach a plateau over the remainder of the space.

## 4.5.4 RMS Slope Validity Criterion

The region of validity of the PO model is seen in Figure 4.6 to be most accurately delineated by a straight line. For Gaussian surfaces, a straight line in the (kh,kl) plane is a line of constant RMS slope. The RMS slope,  $S_{RMS}$ , of a random rough surface can be computed from the second moment of the surface PSD. For



Figure 4.5: Electromagnetic radius of curvature,  $kr_c$ , for Gaussian surfaces over the (kh, kl) parameter space. Lines of constant curvature are shown for  $A_c = 1, 2$ , and 3.

surfaces with a Gaussian PSD,

$$\mathcal{S}_{RMS} = \sqrt{\int K^2 W(K) \mathrm{d}K}$$
(4.40)

$$= \sqrt{2}\frac{h}{l}.$$
 (4.41)

Using the RMS slope, a validity criterion similar to equation (4.1) can be expressed as

$$\mathcal{S}_{RMS} \ll \cos^3 \theta. \tag{4.42}$$

For a more quantitative validity criterion, I include an empirical threshold constant and reformulate the criterion as a relationship between kh and kl:

$$S_{RMS} < A_s \cos^3 \theta$$
  

$$kh < \left(\frac{A_s}{\sqrt{2}} \cos^3 \theta\right) kl.$$
(4.43)



Figure 4.6: dB RMS error values of the PO model for backscatter from surfaces with a Gaussian PSD. The error values are shown for incidence angles of  $0^{\circ}$ ,  $5^{\circ}$ ,  $20^{\circ}$  and  $45^{\circ}$ . The dashed line is the curvature validity condition in equation (4.39) with  $A_c = 2.2$ , and the solid line shows the slope validity condition in equation (4.43) with  $A_s = 0.59$ .

A least-squares fit to the straight line that delineates the 2 dB error values at nadir is computed as  $A_s = 0.59$ . This value is used in conjunction with an angular dependence of  $\cos^3 \theta$  to compute the  $S_{RMS}$  limits at the other incidence angles shown in Figure 4.6.

A comparison of the theoretical  $\cos^3 \theta$  dependence of the PO model and the actual RMS slope cutoff values at various incidence angles is shown in Figure 4.7. The same 2 dB cutoff value described previously is used as the accuracy limit for the PO model. At each integer incidence angle, an empirical fit to the 2 dB error threshold is calculated and converted to a slope value. For incidence angles up to approximately 30°, the  $\cos^3 \theta$  dependence is a good approximation to the empirical limits. For incidence angles greater than 30°, this approximation becomes increasingly inaccurate. Support for the RMS slope validity requirement can be found in the approximations made during the formulation of the PO model. The PO model approximations exclude effects from multiple reflections and shadowing. To fulfill these conditions there must be a combination of sufficiently small surface slopes as well as a grazing angle which exceeds the surface slope [57]. Other indications of the effectiveness of the slope validity criterion are seen in the derivation of the SSA model. As a perturbation expansion about the surface slope such expansions require small slopes as a validity criterion. Because the second order SSA model and PO model are identical formulations, the validity conditions for the two models can be expected to have the same validity criterion with a dependence on slope.



Figure 4.7: RMS slope limits for Gaussian surfaces at which the backscatter error exceeds 2 dB. The theoretical  $\cos^3 \theta$  angular dependence term is shown as a dashed line.

#### 4.5.5 Numerical Results - Power Law Surfaces

From a mathematical perspective, application of the analytic definitions of curvature and surface slope to power law surfaces with  $p \leq 3$  results in infinite curvature and infinite surface slope variance, thus violating the validity criterion in equation (4.1). However, it has been observed empirically that the PO approximation remains valid for many surfaces that violate the curvature criterion [47, 61], including power law surfaces. Warnick, et al. [48] have shown that for near-nadir backscattering from power law surfaces with p = 3, a validity condition for the PO model can be obtained in terms of the significant slope,

$$\S = \frac{h}{L},\tag{4.44}$$

of the surface. At nadir incidence the validity condition becomes  $\S < 0.03$ . The analysis of [48] provides no quantitative information about actual error when this criterion is violated. Because of this, I consider here a condition of the same form as that obtained in [48], but with the upper limit on significant slope to be determined empirically as a function of incidence angle, so that

$$\S < Bf(\theta),\tag{4.45}$$

where B is the significant slope value at nadir and the angular dependence is described by  $f(\theta)$ , f(0) = 1.

Physical optics model error values for power law surfaces are presented in Figure 4.8 for incidence angles of 0°, 5°, 20° and 45° where it can be seen that PO validity criterion is well modeled by a straight line in the (kh, kL) parameter space. From equation (4.44) it can be seen that this a representation of constant significant slope values. As with the Gaussian surfaces, a cutoff value for the accuracy of the PO model is set at 2 dB. A least-squares fit to this error cutoff leads to B = 0.037 in (4.45) at nadir incidence.

It can be seen in Figure 4.8 that the significant slope cutoff decreases as the incidence angle moves away from nadir. Empirically, this variation can be modeled using the same angular dependence as for Gaussian surfaces, so that using



Figure 4.8: dB RMS error values of the PO model for surfaces with a power law PSD. The error values are shown for incidence angles of  $0^{\circ}$ ,  $5^{\circ}$ ,  $20^{\circ}$  and  $45^{\circ}$ . The solid line shows the slope validity condition from equation (4.46).

 $f(\theta) = \cos^3 \theta$  in equation (4.45), the validity condition becomes

$$\S < 0.037 \cos^3 \theta. \tag{4.46}$$

To compare this validity criterion with numerical results, values of the upper bound,  $Bf(\theta)$ , are computed by a least squares fit of (4.45) to the 2 dB error contour in the kh-kL plane at incidence angles from 0-60°. It can be seen in Figure 4.9 that the condition in equation (4.46) matches numerical results well for incidence angles less than 30°.

I note that both (4.46) and the condition (4.43) for Gaussian surfaces involve slope-related quantities and appear as straight lines in the respective parameter spaces. It is important not to read too much into this apparent similarity, because the correlation length l of a Gaussian surface has a very different meaning from the



Figure 4.9: Significant slope values for power law surfaces at which the backscatter error exceeds 2 dB. The cutoff values are computed at integer incidence angles. The theoretical  $\cos^3 \theta$  angular dependence term is shown as a solid line.

dominant wavelength L of a power law surface. For a Gaussian surface, l is on the order of an upper bound on the scale of variation of the surface, whereas L is a lower bound on the spectral content of a power law surface. Similar considerations apply to the difference between RMS slope and significant slope.

#### 4.6 Numerical Results - Other Models

In addition to the PO model, I have investigated the accuracy of other rough surface scattering models. In this section a description of the general accuracy and the region of validity of each model is presented. We show that the validity criteria for many of these models have similar trends to that of the PO approximation due to their relationship with the PO model. Figures 4.10-4.17 are error plots for Gaussian and power law surfaces at 0°, 20°, 45°, and 60° incidence angles of each model. The PO model is included as a reference against which the other models can be compared.

#### 4.6.1 Small Perturbation Method

The accuracy of the SPM model is commonly described by

$$kh \ll 1. \tag{4.47}$$

Because the expansion term of the SPM model includes an angular dependence, another validity criterion of SPM that has been developed requires that [62]

$$kh\cos\theta \ll 1. \tag{4.48}$$

Due to the angular dependence of this term, the region of validity might be expected to increase at larger incidence angles. When applied to Gaussian surfaces, the SPM model shows an accuracy that is limited by the validity criterion in equation (4.47). However, an increase in the region of validity, similar to that described by equation (4.48), can be seen at larger incidence angles when the SPM model is applied to power law surfaces.

I note that the increase in the region of validity is limited to the smaller valid region along the left edge of the plots. A secondary region that can be seen for rougher power law surfaces is caused by a cross-over between the backscatter values of the SPM model and those of the reference solution and cannot be considered to be a true region of accuracy.

#### 4.6.2 Geometrical Optics

The region of validity of the GO model is strongly influenced by incidence angle. For both surface types, the range of accuracy of the GO model is almost entirely limited to angles less than 20°. This occurs because of the non-physically rapid decrease in the magnitude of (4.18) for large incidence angles. The difference in behavior between GO and the reference solution is most evident in figures 4.14-4.17, where the GO model is inaccurate across the entire parameter space for both surface types.



Figure 4.10: dB RMS error values for Gaussian surfaces at 0° incidence angle.

Due to the inseparability of the coherent and incoherent scattering contributions, the GO model shows significant error for values of  $kh \ll 1$  in the near-nadir range where the coherent component is large relative to the incoherent component. This effect is most evident in the Figure 4.13.

## 4.6.3 Monte Carlo Physical Optics

The accuracy of the MCPO model is similar to that of the stochastic PO model. The validity criterion expressed in equation (4.42) and equation (4.46) for the stochastic PO model describes the accuracy of MCPO up to an incidence angle of approximately 20°. As the incidence angle increases the MCPO backscatter values become larger than the reference solution for smoother surfaces.

As with SPM, the accuracy of the MCPO model appears to have a second band of increased accuracy for rougher surfaces. The second band, which is evident



Figure 4.11: dB RMS error values for power law surfaces at 0° incidence angle.

for both surface types, is anomalous, and the MCPO model should not be construed as truly accurate in these regions. These regions of apparent accuracy are especially misleading for the power law surfaces at large incidence angles in Figure 4.15 and Figure 4.17, where the accuracy is no better than the stochastic PO model.

## 4.6.4 Iterated Physical Optics

ITPO is the most accurate model in the study. The region of validity is large for both power law and Gaussian surfaces and demonstrates little variation across the angular range. Application of a 2 dB threshold results in the validity criterion

$$\mathcal{S}_{RMS} < 2.2 - .02\theta, \tag{4.49}$$



Figure 4.12: dB RMS error values for Gaussian surfaces at 20° incidence angle.

for Gaussian surfaces, where  $\theta$  is the incidence angle in degrees and

$$\S < .25 - .002\,\theta,\tag{4.50}$$

for power law surfaces. The limit of the region of validity is sharply defined by a steep decline in accuracy outside the valid region. I note that part of the improved accuracy may be due to use of the same deterministic surfaces as that of the reference solution. However, this effect is probably minor, as indicated by the limited improvement of the deterministic MCPO model over the stochastic PO model.

## 4.6.5 Small Slope Approximation

The fourth order SSA model is the most accurate stochastic model in the study. Because the range of validity of SSA is constant across the angular range of the study, the empirical validity criterion does not include an angular term. The 2 dB



Figure 4.13: dB RMS error values for power law surfaces at 20° incidence angle.

validity criteria for the SSA model are

$$\mathcal{S}_{RMS} < 0.5 \tag{4.51}$$

for Gaussian surfaces and

$$\S < .05$$
 (4.52)

for power law surfaces. Though the PO and second order SSA model are identical, the inclusion of higher order terms in the SSA model increases its range of validity. The increase in accuracy is accompanied by a more sharply defined transition between the accurate and inaccurate regions, and larger error values in the inaccurate region.

## 4.7 Conclusion

I have studied the regions of validity of various rough surface scattering models for surfaces with Gaussian and power law power spectra. Because most of



Figure 4.14: dB RMS error values for Gaussian surfaces at 45° incidence angle.

the models in the study are related to the Kirchhoff approximation, a major focus is placed on the region of validity of the PO approximation.

Existing criteria for the region of validity of the PO model are commonly given in terms of a bound on the curvature of the scattering surface. We found that for backscattering, the range of validity of the PO model for Gaussian surfaces is better described by a bound on the RMS surface slope. Similarly, the validity of the PO model for power law surfaces is accurately modeled by a bound on the significant slope of the surface. Numerical results for both surface types also showed that the region of validity has an angular dependence approximated by  $\cos^3 \theta$  for incidence angles less than 30° and 40° for Gaussian and power law surfaces, respectively.

An important difference between the curvature-based PO validity criterion in equation (4.1) and the significant slope validity criterion developed here is the frequency dependence of the validity criteria. The empirical results indicate that the



Figure 4.15: dB RMS error values for power law surfaces at 45° incidence angle.

surface slope validity criterion is independent of the frequency of the incident wave. It is possible that for larger frequencies than those considered, the region of validity could diverge from equation (4.46) and equation (4.43), although theoretical results in [48] are asymptotic as  $k \to \infty$ .

In addition to the PO model, other models in the study show a correlation between the region of validity and the appropriate surface slope parameter. I note that the deterministic ITPO model is the most accurate model in the study and that the most accurate stochastic model is the fourth order SSA model. A summary of the validity criteria for each model is shown in Table 4.7. From the table, it can be seen that the region of validity of most models is influenced by the incidence angle.



Figure 4.16: dB RMS error values for Gaussian surfaces at 60° incidence angle.

Table 4.1: Validity criteria with a 2dB threshold for PO-based backscattering models in the study. The variables  $S_{RMS}$  and § refer to the Gaussian RMS surface slope and the power law significant slope.

PO Based Model Validity Criteria		
Model	Gaussian Surfaces	Power Law Surfaces
PO	$S_{RMS} < 0.59 \cos^3 \theta$	$\S < 0.037 \cos^3 \theta$
GO*	$S_{RMS} < 0.59 \cos^3 \theta$	$\S < 0.055,  \theta < 25^{\circ}$
MCPO	$S_{RMS} < 0.59 \cos^3 \theta$	$\S < 0.037 \cos^3 \theta$
ITPO	$\mathcal{S}_{RMS} < 2.202 \theta$	$\S < .25002 \ \theta, \theta < 45^{\circ}$
SSA	$S_{RMS} < .5$	$\S < .05$

\*The GO model includes both coherent and incoherent scattering



Figure 4.17: dB RMS error values for power law surfaces at  $60^{\circ}$  incidence angle.

# Chapter 5

# EM Bias Model

### 5.1 Introduction

Operational EM bias models for satellite based altimeters are limited to empirically modeling the bias with parameters that can be estimated from the backscatter characteristics of the surface [6, 7, 8, 9, 10]. These models include the current operational models for the TOPEX/Poseidon and Jason-1 altimeter missions that estimate the bias from significant wave height and wind speed values. Though bias estimates using these parameters are accurate in the mean, a significant amount of variance remains [19].

Theoretical studies of the EM bias have shown that the bias is a composite of two underlying physical mechanisms: non-Gaussian long wave surface statistics and hydrodynamic modulation. The first EM bias model was developed from non-Gaussian ocean surface statistics. Using the weakly non-linear (WNL) theory by Longuet-Higgins [14], the EM bias for a one dimensional surface was described as a function of the skewness and tilt modulation (height-slope cross correlation) of the ocean surface [15]. The contribution of the non-Gaussian statistics was extended to a two-dimensional surface by Srokosz [5]. Further developments were made by Elfouhaily, *et al.* [16] by rederiving the contributions of the WNL theory when applied only to the long wave components of the surface.

The second contributing factor to the EM bias is hydrodynamic modulation. Motivated by a nonlinear surface model of Longuet-Higgins *et al.* [17] which relates the strength of the small wave height modulation to long wave slope, empirical relationships between the slope and the bias have been developed [13, 18, 23]. This approach has been expanded recently by a pair of theoretical models. The first, by Elfouhaily, *et al.*, develops a bias theory that describes the hydrodynamic modulation using the modulation transfer function (MTF) of Alpers and Hasselmann [63]. The second, by Warnick, *et al.* [64], introduces hydrodynamic modulation through the physical optics (PO) scattering approximation. This theory describes the EM bias as a linear function of the RMS wave slope.

The only study to combined non-Gaussian surface statistics and hydrodynamic modulation was a numerical investigation by Rodriguez, *et al.* [27]. In this study, the joint height-slope PDF by Longuet-Higgins [14] and modulation transfer function (MTF) by Alpers and Hasselmann [63] were used to estimate the bias for a one-dimensional surface. The results showed that contributions from the non-Gaussian surface statistics and hydrodynamic modulation are of the same order of magnitude.

In this chapter an EM bias model is developed that includes both non-Gaussian long wave statistics and hydrodynamic modulation. Scattering from small wave surface facets is modeled by the PO scattering approximation. Using the WNL theory, local tilt angles are included in PO-based bias model by Warnick, *et al.* [64]. The result is a bias model that describes the relative EM bias as a sum of contributions from skewness, tilt modulation, and RMS wave slope.

A number of features of the derivation are worth noting. First, an analytical relationship between the MTF and the RMS wave slope is developed that linearly relates the hydrodynamic modulation to the RMS slope. Second, dependencies on small wave roughness and wind speed are included in the EM bias model through the PO model.

The accuracy of the bias estimates is analyzed by a comparison with measured EM bias value from the Gulf of Mexico Experiment (GME). From a model surface spectrum, estimated and measured bias values are compared as a function of wind speed. The EM bias is also evaluated by using measured surface statistics from the GME data set as inputs to the bias theory. Using the measured surface parameters, the contribution from each of the bias terms, hydrodynamic modulation, tilt modulation, and skewness, is investigated. A final analysis shows the residual errors between the bias theory and the measured values.

#### 5.2 EM Bias Model Derivation

The EM bias is defined as the normalized correlation between the surface height,  $\zeta$ , and the backscatter coefficient profile,  $\sigma^{\circ}(\zeta)$ ,

$$\epsilon = \frac{\mathrm{E}\left[\zeta\sigma^{\circ}(\zeta)\right]}{\mathrm{E}\left[\sigma^{\circ}(\zeta)\right]} = \frac{\int \int \zeta\sigma^{\circ}(\zeta,\theta)\mathrm{P}(\zeta,\theta)d\zeta d\theta}{\int \int \sigma^{\circ}(\zeta,\theta)\mathrm{P}(\zeta,\theta)d\zeta d\theta}.$$
(5.1)

This formulation describes the bias as a function of the scattering from small wave surface facets with local incidence angles and heights described by the long wave joint height-slope PDF,  $P(\zeta, \theta)$ .

EM bias measurements by remote sensing instruments are typically divided into three contributing components: skewness bias, EM bias, and tracker bias. The tracker bias is a significant wave height dependent instrumental error that is neglected in this discussion. The remaining biases are inherent in the interaction between the EM signal and the ocean surface, and are often combined as the sea state bias.

In the derivation presented here, the bias seen by the altimeter instrument is shown to be a combination of three bias terms. The skewness bias is related to the skewness of the ocean surface. The tilt modulation is caused by a non-Gaussian distribution of surface slopes along the ocean surface. Last, the hydrodynamic modulation bias created by the modulation of short wave heights with surface displacement. For convenience, since I compute a combination of the skewness and sea state biases, the sum of the three is referred to as the EM bias.

#### 5.2.1 Separation Wavenumber

Because the integral form of the bias in Eq (5.1) includes components of short wave scattering,  $\sigma^{\circ}$ , and long wave statistics,  $P(\zeta, \theta)$ , the division of the ocean surface into long and short wave components is required. The division between the long and short wave components of the ocean spectrum is identified using the separation wavelength,  $\lambda_{sep} = 2\pi/k_{sep}$  such that

$$[\lambda_{\circ} = \mathcal{O}(10m)] \gg \lambda_{sep} \gg \left[\lambda_{em} = \mathcal{O}(10^{-2}m)\right]$$
(5.2)
at microwave frequencies where  $\lambda_{em}$  is the electromagnetic wavelength and  $\lambda_{\circ}$  is the dominant wavelength of the surface spectrum. We note that  $k_{sep}$  influences both the small and long wave portions of the spectrum, and investigate its effect on EM bias estimates in Sec. 5.6.

# 5.2.2 Long Wave Surface Statistics

The joint height-slope distribution of the long wave portion of the ocean surface is derived from the weakly non-linear theory (WNL) [14]. WNL theory describes the surface statistics as a Gram-Charlier expansion in terms of the skewness,  $\lambda_{30}$ , and tilt modulation,  $\lambda_{12}$ , of the ocean surface [5, 16]. The tilt modulation, as used in this work, refers to the cross-correlation of surface height and slope. The long waves are modeled using the long crested assumption so that the slope in one direction is set equal to zero,  $\zeta_y = 0$ . With this approximation the three dimensional joint-height slope PDF is reduced to a two dimensional expression,

$$P(\zeta, \zeta_x) = \frac{e^{-\frac{1}{2}(\eta^2 + \eta_x^2)}}{2\pi h_l s_l} \\ \times \left[ 1 + \frac{\lambda_{30}}{6} \mathcal{H}_{30}(\eta, \eta_x) + \frac{\lambda_{12}}{2} \mathcal{H}_{12}(\eta, \eta_x) \right],$$
(5.3)

where the normalized height and slope are defined as

$$\eta = \zeta/h_l \tag{5.4}$$

$$\eta_x = \zeta_x / s_l, \tag{5.5}$$

and the height and slope variances are  $h_l^2$  and  $s_l^2$ , respectively. The Hermite polynomials,  $\mathcal{H}_{ij}(\eta, \eta_x)$ , used in this paper are

$$\mathcal{H}_{30}(\eta,\eta_x) = \eta^3 - \eta \tag{5.6}$$

$$\mathcal{H}_{12}(\eta, \eta_x) = \eta(\eta_x^2 - 1),$$
 (5.7)

and the explicit dependencies on height and slope are dropped in the remainder of the paper.

## 5.2.3 Short Wave Surface Spectrum

The short wave surface power spectrum is defined by

$$W_s(\mathbf{k}) = \begin{cases} W(\mathbf{k}) & k \ge k_{sep} \\ 0 & k < k_{sep} \end{cases}$$
(5.8)

where  $W(\mathbf{k})$  is the full surface spectrum, and the related correlation function,

$$C(x,y) = \frac{1}{h^2} \int W_s(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{k}$$
(5.9)

is defined as the Fourier transform of the short wave PSD normalized by the surface height variance,  $h_s^{-2}W(\mathbf{k})$ .

With the division of the surface spectrum into long and short wave portions, the small wave power spectrum can be well approximated as a two-dimensional, isotropic power law PSD. By integrating the two-dimensional PSD in angle, the small wave power spectrum can be expressed as

$$W_s(k) = \begin{cases} h_s^2(p-2)k_{sep}^{p-2}k^{1-p} & k \ge k_{sep} \\ 0 & k < k_{sep}, \end{cases}$$
(5.10)

where p is the exponent of the power law spectrum. Because the small wave PSD is isotropic, the two-dimensional correlation function can be expressed as a function of distance,  $\rho = (x^2 + y^2)$ , such that

$$C(z) = (p-2)z^{p-2}2^{1-p}\frac{\Gamma\left(1-\frac{p}{2}\right)}{\Gamma\left(\frac{p}{2}\right)} + {}_{1}\mathcal{F}_{2}\left(\left[1-\frac{p}{2}\right], \left[1,2-\frac{p}{2}\right], -\frac{z^{2}}{4}\right)$$
(5.11)

where  $p \neq 3$ . The symbol  $_{1}\mathcal{F}_{2}$  refers to the hypergeometric function and the substitution  $z = k_{sep}\rho$  is used to obtain the final expression. A similar correlation function can be developed for p = 3. For the remainder of the paper, the explicit dependencies of the correlation function are dropped so that C = C(z).

Note that the approximation of the surface spectrum as a pure power law form is used only for the small wave portion of the spectrum. This allows the bias model to encompass realistic sea surface states which can deviate significantly from an averaged model that constrains the form of the spectrum over the full range of long and short waves. The description of the small wave spectrum as a two-dimensional, isotropic PSD creates an effective surface model that is composed of two-dimensional small wave surface facets on a corrugated, or one-dimensional, long wave surface.

#### 5.2.4 Hydrodynamic Modulation

The modulation of short wave heights as a function of surface displacement causes a differential in surface roughness between the crests and troughs of the ocean surface. This results in larger specular returns from the troughs, contributing to a differential in returned power that is a source of the EM bias. This contribution to the EM bias is referred to as the hydrodynamic bias.

Modeling the hydrodynamic modulation of the ocean surface is commonly accomplished by using a modulation transfer function, such as that developed by Alpers and Hasselmann [63]. The MTF describes the modulation of the surface by an interaction of the long and short wave surface components. An example of the change in the small wave height variance,  $\Delta h_s^2$ , is shown in Figure 5.1 for a typical ocean surface. It is apparent that the dependence of  $\Delta h_s^2$  is approximately linear with surface displacement. Similar results have been reported in numerical studies and experimental measurements [27, 64]. Approximating the small wave height as a linear function of surface displacement results in the expression

$$h(\zeta) = h_{\circ} \left( 1 + m \frac{\zeta}{h_l} \right) \tag{5.12}$$

where m is a modulation coefficient. By using this expression in the EM bias model, the effects of hydrodynamic modulation are included. In Sec. 5.3.1 the MTF is used to relate m to long wave statistics.

#### 5.2.5 Physical Optics Scattering Model

The most common scattering model used in EM bias theories is the geometrical optics (GO) model. Geometrical optics uses the infinite frequency, or ray tracing, approximation to model scattering from a rough surface. With this approximation, surfaces are considered smooth with respect to the incident EM wavenumber,



Figure 5.1: Application of the modulation transfer function (MTF) to surface described by the unified ocean spectrum by Elfouhaily, *et al.* [4] shows a linear correlation between the magnitude of the short wave modulation and the normalized surface height.

thus applying an inherent high wavenumber cutoff to the surface spectrum. Because the modulated small wave heights are on the same order of magnitude as the incident wavenumber, the GO approximation does not accurately model small wave effects such as hydrodynamic modulation.

A more accurate description of rough surface scattering is provided by the physical optics (PO) scattering model [57, 60]

$$\sigma^{\circ}(\theta) = \frac{k_{em}^2 \cos^2 \theta}{4\pi} \int \int e^{ik_b x} e^{-\lambda(1 - C(x,y))} dx dy, \qquad (5.13)$$

where  $\theta$  is the incidence angle, the Bragg wavenumber,  $k_b = 2k_{em}\sin\theta$ , and  $\lambda = (2k_{em}h\cos\theta)^2$ . In contrast to the GO approximation, the PO model requires as inputs the surface height variance,  $h^2$ , and the correlation function, C(x, y). Because the PO scattering model is applied to small wave facets of the surface, the height variance used in the model is the short wave height variance,  $h_s^2$ .

The accuracy of the PO scattering model for power law surfaces has been addressed by two recent studies. The first, develops a theoretical validity criterion of the PO model for two-dimensional power laws surfaces [48]. The numerical scattering study in Chapter 4 shows that at near-nadir incidence angles, the validity criterion for the PO model is satisfied for ocean-like surfaces with a power law PSD, as long as the significant slope is less than 0.03.

#### 5.2.6 Backscatter Coefficient Profile

An analytic expression for the backscatter coefficient profile,  $\sigma^{\circ}(\zeta)$ , is created from expressions for the long wave joint height-slope PDF, (5.3), and the radar cross section, described by the PO model, (5.13). By using the approximations for the correlation function, (5.11), and hydrodynamic modulation, (5.12), in the PO model the resultant expression,

$$\sigma^{\circ}(\zeta) = \int d \tan \theta \, \frac{k_{em}^2 \cos^2 \theta}{4\pi}$$

$$\times \int \int dx \, dy e^{i2k_{em} \sin \theta} e^{-\lambda(\eta)(1-C)}$$

$$\times \frac{1}{2\pi h_l s_l} e^{-\frac{1}{2} \left[ \left(\frac{\zeta}{h_l}\right)^2 - \left(\frac{\tan \theta}{s_l}\right)^2 \right]}$$

$$\times \left( 1 + \frac{\lambda_{30}}{6} \mathcal{H}_{30} + \frac{\lambda_{12}}{2} \mathcal{H}_{12} \right),$$
(5.14)

is a three-dimensional integral that appears in both the numerator and the denominator of the EM bias definition, (5.1).

To create an analytical solution, small local incidence angles are assumed such that

$$\sin\theta \approx \theta \tag{5.15}$$

$$\cos\theta \approx 1 \tag{5.16}$$

$$\tan\theta \approx \theta, \qquad (5.17)$$

and the order of integration is changed so that the first integration performed is over the long wave slopes,

$$\sigma^{\circ}(\zeta) = \frac{k_{em}^{2} e^{-\eta^{2}/2}}{4\pi\sqrt{2\pi}} \int \int dx \, dy e^{-\lambda(\eta)(1-C)}$$

$$\times \int d\eta_{x} e^{i\mu\eta_{x}} \frac{e^{-\eta_{x}^{2}/2}}{\sqrt{2\pi}} \left(1 + \frac{\lambda_{30}}{6} \mathcal{H}_{300} + \frac{\lambda_{12}}{2} \mathcal{H}_{120}\right)$$
(5.18)

where  $\mu = 2xk_{em}s_l$ . The integral over the local tilt angles can be solved analytically with the identities

$$\int e^{i\mu x} x^2 e^{-x^2/2} dx = \sqrt{\frac{\pi}{2}} e^{-\mu^2/2} (1-\mu^2)$$
(5.19)

$$\int e^{i\mu x} e^{-x^2/2} dx = \sqrt{2\pi} e^{-\mu^2/2}, \qquad (5.20)$$

and a simplified expression for equation (5.19) can be written as

$$\sigma^{\circ}(\zeta) = \frac{k_{em}^2 e^{-\eta^2/2}}{4\pi\sqrt{2\pi}} \int \int dx \, dy \, e^{-\lambda(\eta)(1-C)} e^{-\mu^2/2} \\ \times \left(1 + \frac{\lambda_{30}}{6} \left(\eta^3 - \eta\right) - \frac{\lambda_{12}}{2} \eta \mu^2\right).$$
(5.21)

The expression for the backscatter coefficient profile is now reduced to a two dimensional integral in the x and y dimensions.

# 5.2.7 EM Bias

The final expression of the EM bias model is created by expanding the bias definition in equation (5.1) in a power series about  $\zeta = 0$ ,

$$\epsilon = \frac{E\left[\zeta\left(\sigma^{\circ}(0) + \zeta\sigma^{\circ\prime}(0) + \ldots\right)\right]}{E\left[\sigma^{\circ}(0) + \zeta\sigma^{\circ\prime}(0) + \ldots\right]},\tag{5.22}$$

where  $\sigma^{\circ'}(0)$  refers to the derivative of  $\sigma^{\circ}(\zeta)$  with respect to  $\zeta$ . Justified by results from Warnick, *et al.* [64], the average backscatter coefficient is approximately linear with displacement, and the expression in equation (5.22) can be truncated after the second term. Using this approximation, equation (5.22) reduces to

$$\epsilon \approx h_l^2 \frac{\sigma^{\circ'}(0)}{\sigma^{\circ}(0)},\tag{5.23}$$

where

$$\sigma^{\circ\prime}(0) = \frac{-1}{\sqrt{2\pi}} \int \int dx \, dy e^{-(2kh_o)^2(1-C)-\mu^2/2} \times \left[ 8m(kh_o)^2 \left(1-C\right) + \frac{\lambda_{30}}{6} + \frac{\mu^2 \lambda_{12}}{2} \right],$$
(5.24)

and

$$\sigma^{\circ}(0) = \frac{1}{\sqrt{2\pi}} \int \int dx \, dy e^{-(2kh_o)^2 (1-C)} e^{-\mu^2/2}.$$
 (5.25)

The notation can be simplified by writing the bias expression as

$$\epsilon = -H(\gamma m + \kappa \lambda_{30} + \tau \lambda_{12}). \tag{5.26}$$

where the significant wave height,  $H = 4h_l$ , and the coefficients  $\gamma$ ,  $\kappa$ , and  $\tau$  are

$$\gamma = \frac{\int \int 2k_{em}^2 h_o^2 (1-C) e^{-(2k_{em}h_o)^2 (1-C)} e^{-\mu^2/2} \, dx \, dy}{\int \int e^{-(2k_{em}h_o)^2 (1-C)} e^{-\mu^2/2} \, dx \, dy}$$
(5.27)

$$\tau = \frac{1}{8} \frac{\int \int \mu^2 e^{-(2k_{em}h_{\circ})^2 (1-C)} e^{-\mu^2/2} \, dx \, dy}{\int \int e^{-(2k_{em}h_{\circ})^2 (1-C)} e^{-\mu^2/2} \, dx \, dy}$$
(5.28)

$$\kappa = \frac{1}{24}.\tag{5.29}$$

A review of the final bias model in equation (5.26) shows the leading significant wave height dependence, common to all bias models, followed by terms relating the EM bias to the surface skewness, tilt modulation, and hydrodynamic modulation. It can also be seen that the the dependence of the EM bias on  $\lambda_{12}$  and  $\lambda_{30}$  is similar to the results from models by Elfouhaily, *et al.* [16] and Srokosz [5]. An analysis of the terms and coefficients that describe the model is conducted in subsequent sections.

## 5.3 Bias Coefficients

The dependence of the bias on the short wave spectrum is carried in the coefficients  $\gamma$ ,  $\kappa$ , and  $\tau$ . As seen in equation (5.26), the short wave coefficients and the long wave surface parameters combine to create the individual bias terms. It should be noted that the coefficients  $\tau$  and  $\kappa$  have been present in previous bias models, but with modifications in this model due to the PO approximation. The bias coefficient  $\gamma$  is a result of the inclusion of hydrodynamic modulation and is investigated in Warnick, *et al.* [64], but without long wave tilting effects.

#### 5.3.1 Hydrodynamic Modulation Coefficient

In a previous EM bias model by Warnick, *et al.* [64] the value of the hydrodynamic modulation coefficient,  $\gamma$ , is shown to asymptotically approach 1/(p-2) for large values of  $k_{em}h$ . Though that model do not include the long wave tilting of short wave scattering facets, Figure 5.2 shows that the effect of the long wave tilting is negligible in the asymptotic region. It should also be noted that typical values of the surface are such that  $k_{em}h > 1$ , and  $\gamma$  is well approximated by the asymptotic limit of 1/(p-2). Because the value  $\gamma$  is in the asymptotic region, changes in  $k_{em}h$  do not have a significant effect on  $\gamma$ , and the hydrodynamic bias is not strongly frequency dependent.

#### 5.3.2 Tilt Modulation Coefficient

The inherent peakedness of ocean waves causes fewer horizontal facets to be present near the crests than the troughs, thus introducing a tilt modulation bias. For a surface that is smooth on the order of the incident EM wavelength, the tilt



Figure 5.2: Values of the hydrodynamic modulation coefficient,  $\gamma$ , as a function of the electromagnetic height of the small waves,  $k_{em}h_s$ . For realistic ocean surfaces,  $k_{em}h_s > 1$  so that  $\gamma$  is not greatly affected by changes in small wave heights.

modulation results in a larger specular return from the troughs than the crests. By using the GO approximation, the model by Srokosz [5] inherently uses the smooth surface approximation, resulting in a tilt modulation bias described by

$$\beta_{tilt} = -\frac{1}{8}\lambda_{12} \tag{5.30}$$

where the tilt modulation coefficient,  $\tau$ , is a constant 1/8.

The contribution of the tilt modulation bias is modified with the inclusion of small scale roughness in the scattering model. Figure 5.3 shows the value of  $\tau$  as a function of the electromagnetic height,  $k_{em}h_s$ . As  $k_{em}h_s$  approaches 0, the surface becomes more smooth, and the PO result asymptotically approaches the GO value of  $\tau = 1/8$ . For rougher surfaces, the surface scattering becomes more Lambertian, and the effect of local tilt angles decreases. For an infinitely rough surface  $(k_{em}h_s \to \infty)$ the scattering is perfectly isotropic, and the tilt modulation bias vanishes.

Changes in the value of  $\tau$  with respect to  $k_{em}h_s$  also indicate a frequency dependence of the tilt modulation bias. An increase in  $k_{em}h_s$  can also be interpreted as an increase in the incident EM frequency, so that the tilt modulation bias seen in Figure 5.3 decreases with incident wavenumber. Because  $\tau$  is the only term that changes significantly with  $k_{em}h$ , the frequency dependence of the model is wholly contained in the tilt modulation coefficient.

#### 5.3.3 Skewness Coefficient

From (5.29), the skewness bias coefficient has a constant value of  $\kappa = 1/24$ . This result is identically equal to results from previous models by Srokosz [5] and Elfouhaily, *et al.* [16].

#### 5.4 Hydrodynamic Modulation and Slope

A number of bias models have been developed using an empirical relationship between measured RMS slope values and the EM bias [18, 23]. These models are implicitly based on the relationship between wave slope and the modulation coefficient developed by Longuet-Higgins [17]. The first bias work to explicitly state this relationship in the context of the EM bias is by Melville, *et al.* [13]. By using a two



Figure 5.3: Values of the tilt modulation coefficient as a function of electromagnetic height of short ocean waves,  $k_{em}h_s$ . For smooth surfaces,  $\tau = 1/8$ , as derived by Srokosz [5]. An infinitely rough surface would have no tilt modulation bias. Typical values of  $k_{em}h_s$  are greater than 1, so that  $\tau < 1/8$ .

frequency surface model, the modulation coefficient and RMS wave slope are shown to be equal to first order. Warnick, *et al.* [64] uses the same relationship and finds that the RMS wave slope is strongly correlated to the bias. Here we develop a more rigorous relationship between the modulation coefficient, m, and the RMS long wave slope, S.

## 5.4.1 Modulation Transfer Function

Height modulation of the short waves is described by a modulation transfer equation (MTF) developed by Alpers and Hasselmann [63]. The MTF describes the height modulation of the short waves as a function of spectral frequency,

$$\delta W(k_s) = W(k_s) \int dk_l \, z(k_l) R(k_s, k_l) \mathrm{e}^{i(k_l x - \omega_l t)},\tag{5.31}$$

where  $z(k_l)$  is the Fourier transform of the long wave profile,  $\zeta(x, t)$ , and  $\omega_l$  and  $k_l$  are the long wave angular frequency and wavenumber, respectively. The one dimensional form of the modulation transfer function can be written as

$$R(k_s, k_l) = -k_l \frac{\omega_l + i\mu_s}{\omega_l^2 + \mu_s^2} \frac{c_l}{c_s} \left[ \frac{1}{F(k)} \frac{\partial F(k)}{\partial k} - \frac{\gamma_s}{k_s} \right] \\ \times \left( c_s k_l k_s - \frac{1}{2} k_l \omega_l \right)$$
(5.32)

where  $\mu_s$  is the short wave relaxation rate,  $c_l$  and  $c_s$  are the long wave and short wave phase speeds,

$$\gamma_s = \frac{1}{2} \frac{1 + 3(\tau_w k_s^2 / \rho_d g)}{1 + \tau_w k_s^2 / \rho_d g},$$
(5.33)

and the constants  $\tau_w = 74 \times 10^{-6} \text{ m}^3/\text{s}^2$  and  $\rho_d = 1027 \text{ kg/m}^3$  are the surface tension and density of water.

From the MTF, the change in the local small wave height variance can be described as

$$\Delta h_s^2 = \int \delta W(k_s) dk_s, \qquad (5.34)$$

so that the total small wave height variance is

$$h_s^2 = h_o^2 + \Delta h_s^2.$$
 (5.35)

With this definition, the surface height variance can be computed from instantiations of a surface profile. It is shown in Figure 5.1 that  $h_s$  computed from instantiating the surface profile is approximately linear with surface displacement, so that  $h_s$  can described by equation 5.12.

#### 5.4.2 Modulation Coefficient and RMS Slope

To develop a relationship between m and S, I have approximated the ocean surface using independently varying long and short wave power law spectra. Developing the relationship in this way has two important implications. First, by using simplified power law PSDs to describe the surface, an analytic relationship between the modulation coefficient and RMS wave slope can be obtained in terms of long wave surface parameters. We emphasize that this simplified model for the long wave PSD is used only to develop an analytic equation that approximates the relationship between m and S. The second important assumption is the description of the surface as two independent spectra. This separation of long and short wave surface spectra allows for changes in environmental conditions that may vary widely from average values described by a surface spectral model. By varying the long and short waves independently, the first order dependence between the modulation and surface slope can be more clearly discerned.

The long wave surface PSD is described by

$$W(k) = \begin{cases} h_l^2(p-1)k_{min}^{p-1}k^{-p} & k \ge k_{min} \\ 0 & k < k_{min}. \end{cases}$$
(5.36)

and the small wave PSD described in equation (5.10). The exponent p = 3 has been used as a typical value for the ocean surface, while similar results can be obtained for other values of the exponent.

Physically, the tilt modulation is described as the normalized correlation between the wave height squared,  $h_s^2$ , and surface displacement,  $\zeta$ . From the MTF, Rodriguez *et al.* [27] shows that the correlation of  $h^2$  and  $\zeta$  can be expressed as

$$E \left[h^{2} \zeta\right] = \int_{k_{sep}}^{\infty} dk_{s} W(k_{s})$$
  
 
$$\times \int_{k_{min}}^{k_{s}/5} dk_{1} W(k_{1}) \Re \left\{R(k_{s}, k_{1})\right\}.$$
(5.37)

This equation can be reduced to an analytic expression in terms of  $h_l$  and  $k_{min}$  with a few simplifying assumptions. First, the modulating wavenumbers are approximated by the deep wave dispersion relation

$$k_l = \frac{\omega_l^2}{g}.\tag{5.38}$$

By approximating  $\gamma \approx 1/2$  in the MTF in equation (5.33) and noting that  $\frac{1}{2}k_l\omega_l \ll c_s k_l k_s$  and  $\mu_s \ll \omega_l$ , the short wave modulation can be expressed as

$$E\left[h^{2}\zeta\right] \approx -\frac{7}{2} \int_{k_{sep}}^{\infty} dk_{s} W(k_{s}) \int_{0}^{k_{s}/5} dk_{l} \, k_{l} W(k_{l}), \qquad (5.39)$$

where  $W(k_s)$  and  $W(k_l)$  are defined in equation (5.10) and equation (5.36), respectively. By carrying out the integrations, and using the definition of m in equation (5.12), an expression for the correlation coefficient can be written as

$$m = \frac{E[h^2\zeta]}{2h_l h_s^2} \tag{5.40}$$

$$\approx \frac{7}{2}h_l k_{min},\tag{5.41}$$

with the normalization factor,  $1/(2h_lh_s^2)$ . The result shows that for a power law surface the leading dependence of the modulation coefficient is linear with  $h_l$  and  $k_{min}$ .

As an aside, it should be noted that  $h_l k_{min}$  is the significant slope of a power law surface and could be used directly for an ocean surface with an ideal power law surface spectrum. Due to variations from the ideal in ocean surface PSDs, the RMS wave slope is a more accurate measure of actual ocean conditions. In addition, the empirical correlation between the bias and RMS slope noted in previous models motivates its use. A relationship between m and S is developed by using the definition of the RMS surface slope,

$$S = \sqrt{\int_{k_{min}}^{k_{sep}} dk \, k^2 W(k)}.$$
 (5.42)

By evaluating the integral using the spectrum in equation (5.36), the RMS wave slope can be expressed as

$$S = h_l k_{min} \sqrt{2 \ln\left(\frac{k_{sep}}{k_{min}}\right)},\tag{5.43}$$

where the leading dependencies of S are linear with the standard deviation of the height,  $h_l$ , and the minimum wavenumber,  $k_{min}$ . An additional dependence on  $k_{min}$  can also be seen under the square root. However, this dependence is a slowly varying function of the minimum wavelength and can be approximated as a constant for  $k_{min} \ll k_{sep}$ .

The resulting dependencies of m in equation (5.41) and S in equation (5.43), on  $h_l k_{min}$  are to first order related by a constant multiple. Figure 5.4 shows m and S computed from the analytic solutions along with the value of mcomputed from the MTF in equation (5.37).

The results of this numerical study verify the connection between m and S seen in previous empirical models [18, 13, 23]. By recognizing the linear correlation between slope and modulation coefficient such that

$$\nu = m/S,\tag{5.44}$$

the RMS wave slope can be used as a proxy variable for the hydrodynamic modulation coefficient. By establishing a relationship between the long wave RMS slope and the modulation coefficient, the EM bias can be rewritten as

$$\epsilon = -H(\gamma\nu S + \kappa\lambda_{30} + \tau\lambda_{12}), \qquad (5.45)$$

where contributions from the long wave parameters, S,  $\lambda_{30}$ , and  $\lambda_{12}$  and short wave coefficients  $\gamma$ ,  $\tau$ , and  $\lambda$  are identifiable.

From the near linear relationship of m and S seen in Figure 5.4, it is apparent that  $\nu$  is, to a large degree, independent of the long wave statistics, and that



Figure 5.4: The near linear relationship between the modulation coefficient, m, and the RMS wave slope, S. This relationship is used to approximate  $\nu = m/S$  as a constant when computing bias values from the GME data set in Sec. 5.7.

changes in swell or other long wave effects impact S and m equally. This description of the value for  $\nu$  is used to compute the bias model with the GME data set.

The value of  $\nu$  can also be computed directly from a surface spectral model by using equation (5.42) to compute the RMS slope and the MTF to compute m. Computing  $\nu$  from a model spectrum includes the effects of changing short and long wave spectra together and an additional weak dependence on the wind speed through the frictional velocity. The results of this method are seen in Figure 5.5.

## 5.5 Long Wave Parameters

The three contributing factors to the EM bias are dependent on the long wave parameters S,  $\lambda_{12}$ , and  $\lambda_{30}$ . In this section I describe the methods to compute these values from the surface PSDs.



Figure 5.5: The value of  $\nu$  computed from a spectral model. Variations in the value as a function of wind speed are reflective of simultaneous changes in the spectral model to both the long and short wave portions of the spectra.

Using the WNL theory, previous studies have derived expressions for  $\lambda_{12}$ and  $\lambda_{30}$  as integrals of the surface PSD. By applying these expression to a model surface spectrum, values of S,  $\lambda_{12}$ , and  $\lambda_{30}$  can be computed as a function of wind speed. These expressions are also applied to measured surface spectra from the GME data set to compute the long wave parameters.

#### 5.5.1 RMS Wave Slope

Values of the RMS wave slope are computed from measured surface profiles using the statistical definition of the RMS wave slope in equation (5.42). Applying this method to the GME data results in the slope values shown in Figure 5.6, where slope is plotted against measured wind speeds. Using the same computational method, the slope as a function of wind speed is shown as computed from the unified surface spectral model.

Due to averaging across the footprint, slope measurements profiles from the GME data underestimate the RMS slope. A comparison of slope measurements relates radar and laser slope measurements as

$$S_{Laser} = 1.11S_{Radar} + 0.014. \tag{5.46}$$

To compensate, slope measurements from GME data set are scaled to reflect this relationship.

## 5.5.2 Tilt Modulation

The tilt modulation is a measure of the correlation between height and local tilt angle defined by

$$\lambda_{12} = \frac{E\left[\zeta\zeta_x^2\right]}{E\left[\zeta\right]^{1/2}\left[E\zeta_x\right]}.$$
(5.47)

From the WNL theory, Jackson [15] derives an expression for the total tilt modulation as a function of the surface PSD such that

$$\lambda_{12} = \frac{4}{h_l s_l^2} \int_0^\infty dk W(k) \int_0^k dl \left(2k^2 l + l^3\right) W(l).$$
 (5.48)

Because the value of interest for the EM bias model is the long wave tilt modulation, the upper limits of integration in equation (5.48) are changed to  $k_{sep}$  to include only



Figure 5.6: Long wave RMS slope values as a function of wind speed. The lines show slope values computed from the unified surface spectrum by Elfouhaily, *et al.* [4]. The scatter plots show RMS slope values of the GME data set computed from the surface PSD and equation (5.42)



Figure 5.7: Long wave tilt modulation values as a function of wind speed. The lines show tilt modulation values computed from the unified surface spectrum by Elfouhaily, *et al.* [4]. The scatter plot shows tilt modulation values of the GME data set computed from the surface PSD and equation (5.48).

the long surface waves when computing values for  $\lambda_{12}$ . After changing the limits of integration, values of the long wave portion of the unified spectrum are shown in Figure 5.7. Values of the tilt modulation from the GME data set are shown in the same figure.

#### 5.5.3 Skewness

The long wave skewness is defined by

$$\lambda_{30} = \frac{E[\zeta^3]}{E[\zeta]^{3/2}}.$$
(5.49)

From WNL theory the full wave skewness is described by Jackson [15] and Longuet-Higgins [14] as the integral equation

$$\lambda_{30} = \frac{12}{h_l^3} \int_0^\infty dk W(k) \left[ \int_0^k dl \, l W(l) \right].$$
 (5.50)



Figure 5.8: Long wave skewness values as a function of wind speed. The lines show skewness values computed from the unified surface spectrum by Elfouhaily, *et al.* [4]. The scatter plot shows skewness values of the GME data set computed from the surface PSD and equation (5.50).

Skewness values computed from surface PSDs of the unified spectrum and GME data are shown in Figure 5.8.

By assuming ergodicity of the surface profile, the surface skewness can be computed directly from the measured surface profiles. As seen in Figure 5.8, values of the skewness coefficient computed directly from the GME profiles are significantly different from measurements made from the PSDs. Because direct measurements of the skewness values are small, the skewness bias will be neglected when computing bias values.

# 5.6 EM Bias Using Spectral Model for Long Wave Parameters

In this section estimates of the EM bias are computed as a function of wind speed and compare the estimated values with measurements from the GME data set.



Figure 5.9: Description of the parameters and coefficients used to compute the estimated EM bias values from the unified surface spectrum [4].

Following the diagram in Figure 5.9, long wave parameters and bias coefficients are computed from the unified surface spectrum as a function of wind speed and inverse wave age. The short wave surface spectrum used to compute the bias coefficients is approximated by fitting a power law spectrum to the tail of the unified surface spectrum.



Figure 5.10: Effect of the separation wavenumber,  $k_{sep}$ , on EM bias values for  $k_{sep} = \pi/2$ ,  $\pi$ ,  $3\pi/2$ , and  $2\pi$ .

# 5.6.1 Separation Wavenumber

To compute the EM bias, a separation wavenumber between the long and short wave spectra must be established. From equation (5.2), the separation wavelength,  $\lambda_{sep}$ , is typically on the order of one meter. Variations in the EM bias model, (5.45), as a function of  $k_{sep}$  are shown in Figure 5.10 for separation wavelengths between  $\lambda_{sep} = 1$  m and  $\lambda_{sep} = 4$  m.

For *in situ* measurements the separation wavelength is constrained to be greater than the spatial resolution, or spot size, of the radar. Calculations of the long wave parameters for the GME data are done for separation wavelength of 2.5 m to avoid high frequency distortion in the surface profiles. I use the same value when computing bias values in the theory so that  $k_{sep} = 4\pi/5$ .



Figure 5.11: The magnitude of the EM bias decreases as a function of the incident wavenumber showing the frequency dependence of the EM bias. The frequency dependence is almost entirely due to the change in the tilt modulation coefficient with  $k_{em}$ .

# 5.6.2 Frequency Dependence

Variation of the EM bias with electromagnetic frequency has been noted in a number of experiments. It is seen in Figure 5.3, that the frequency dependence is primarily due to changes in the tilt modulation bias with incident wavenumber. A typical example of this frequency dependence is shown in Figure 5.11 where values of the bias are calculated from the spectral model using U = 7 m/s and  $\Omega = 1$ . In the following sections, estimated bias values are computed for an incident wavenumber of  $k_{em} = 100$  rad/m.



Figure 5.12: EM bias values computed using a spectral model to get long wave parameters as a function of wind speed and inverse wave age. The points are *in situ* EM bias measurements from the GME data set.

# 5.6.3 Wind Speed Dependence

Estimates of the EM bias using a surface spectral model are shown in Figure 5.12 as a function of the wind speed. On the same plot are measured bias values from the GME data set. Differences between the estimated and measured bias values are partly due to differences between the measured PSDs and the spectral model. Using the spectral model to estimate surface parameters results in significant wave heights that increase exponentially with wind speed. This growth is reflected in the exponential growth of the bias estimates. Measured significant wave height values from the GME data set do not reflect this same relationship. Instead it can be seen that swell is present at low wind speeds, and the growth of measured wave heights is smaller for the measured values than predicted by the spectral model. These differences in H are cause of the primary difference in bias estimates seen in the figure.



Figure 5.13: Relative EM bias,  $\beta$ , values computed with long wave parameters obtained from the unified surface spectrum of Elfouhaily, *et al.* [4] as a function of wind speed and inverse wave age. The points are *in situ* relative bias measurements from the GME data set.

Figure 5.13 shows estimated and measured values of the relative bias as a function of wind speed. The large discrepancies in relative bias values that are apparent in the plot are caused by differences in the PSDs. Differences between the value are reflected in larger values of significant wave height, slope, and tilt modulation, than measured values at the same wind speed. Of particular note are the tilt modulation values that estimated from WNL theory, but have not been measured directly to verify the accuracy of such estimates.

It can be seen that estimates of the EM bias are not shown in Figures 5.12 and 5.13 for wind speeds less than 2 m/s. Because the dominant wavelength of the unified surface spectrum is dependent on wind speed, at small wind speeds the value of the separation wavelength is less than the dominant wavelength. When this occurs the long wave height variance,  $h_l$ , approaches zero, and the results of the model are non-physical, and are therefore not shown.

## 5.7 EM Bias Using Measured Long Wave Parameters

To assess the validity of the bias theory, estimated C-band bias values computed using measured long wave surface parameters are compared with measured bias values. The contribution of the individual bias terms is also described.

#### 5.7.1 Estimating the Bias

In addition to the long wave surface parameters, computed from the GME data set, the short wave coefficients must be approximated to estimate the bias. Coefficient values for each data point are computed from the wind speed measurement and the spectral model, assuming a value of  $\Omega = 1$ . A constant value of the coefficients is assumed such that  $\gamma = 0.42$  and  $\tau = 0.08$ . The value of  $\nu$  is also set to a constant value of 0.79, reflective of the relationship between m and S in Figure (5.4).

A time series of the bias terms, shown in Figure 5.14, shows that the EM bias is composed of almost equal contributions from the hydrodynamic bias,  $\beta_{hydro} = -\gamma \nu S$  and  $\beta_{tilt} = -\tau \lambda_{12}$ . Because the skewness bias,  $\beta_{skew} = -\kappa \lambda_{30}$ , is limited to values of small magnitude with an average close to zero, its contribution to the bias can be neglected. It is also apparent that RMS wave slope and tilt modulation are strongly correlated so that a two dimensional model of the bias as a function of S and  $\lambda_{12}$  is unstable. The instability can be seen when comparing the contour plot of the bias theory, seen in Figure 5.15, and the least-squares model computed from the GME data set, in Figure 5.16. Because of the instability, inherent in the data, the values in the unshaded region are similar, but values in the shaded region can be seen to have significant differences. It is assumed from the form of the model that the contribution of the RMS slope and tilt modulation should be complimentary, but not identical. To develop a better understanding of the EM bias direct measurements of  $\lambda_{12}$  are needed.

A direct comparison of the measured and estimated bias values is shown in Figure 5.17, with the measured values shown as a solid line. The difference between



Figure 5.14: Values of the hydrodynamic modulation bias,  $\gamma \nu S$ , and tilt modulation bias,  $\tau \lambda_{12}$ .



Figure 5.15: Two dimensional fit to the GME data of the relative bias measurements as a function of RMS slope and tilt modulation. Due to the strong correlation between S and  $\lambda_{12}$ , the model is unstable, so that the least-squares fit is not considered valid outside the unshaded region.



Figure 5.16: Two dimensional model of the relative bias from the bias theory with  $\gamma = 0.42$ ,  $\nu = 0.79$ , and  $\tau = 0.08$ . The unshaded region indicates the data region of the GME data set.



Figure 5.17: Comparison of measured and estimated relative bias values from the GME data set. Measured bias values are shown as the solid line, and estimated values are shown as a dashed line.

measured and estimated bias values is shown in Figure 5.18 where the estimated bias shown in the top axis is  $\beta_{hydro}$  and the bias in the lower axis is  $\beta_{hydro} + \beta_{tilt}$ . RMS error values between the estimated and measured values are  $\sigma_{hydro} = 2.09$  (%H) and  $\sigma_{hydro+tilt} = 0.78$  (%H).

## 5.8 Discussion

This paper discusses the derivation and validity of an EM bias model developed using the physical optics scattering model, hydrodynamic modulation, and non-Gaussian long wave statistics. The final form of the model expresses the relative bias as a function of long wave surface parameters. Coefficients of the long wave parameters are computed from the short wave surface spectrum so that effects of both long and short ocean waves are included in the EM bias theory.



Figure 5.18: Differences between measured and estimated relative bias measurements. The top axis shows  $\beta_{hydro} = \gamma \nu S$ , and the lower axis is  $\beta_{hydro} + \beta_{tilt}$ . The contribution from  $\beta_{tilt}$  can be seen to improve bias estimates over just  $\beta_{hydro}$ .

The relative bias is shown to be a sum of three bias values, the hydrodynamic bias, the tilt modulation bias, and the skewness bias, that are each described as the product of a long wave surface parameter and a short wave coefficient. Expressions for the tilt modulation coefficient and skewness coefficient have been described in previous models and are similar to expressions derived here. Differences in the expressions between this model and previous models are due to the use of the physical optics (PO) scattering approximation, as opposed to the more common geometrical optics (GO) approximation. By using the PO approximation, changes in the EM bias as a function of the small wave spectrum can be explained in terms of the physics related to rough surface scattering from small wave facets. The inclusion of the PO approximation also emphasizes the influence of scattering from ocean waves that are of the same order of magnitude as the incidence EM wavenumber, where variations in these waves are shown to cause of the frequency dependence of the bias.

Using a numerical study of the modulation transfer function, the hydrodynamic modulation of short wave heights is shown to be linearly related to surface displacement through the modulation coefficient, m. By including the modulation coefficient in the derivation of the bias theory, the effects of hydrodynamic modulation are included.

The relationship of m with the RMS wave slope, S, is also investigated. Using a simplified spectral model for the ocean surface, m and S are shown to be nearly linearly related. This relationship has been used implicitly in previous models that have described the bias as a function of wave slope, and provides a physical mechanism to describe the relationship between the bias and the RMS wave slope.

As a function of wind speed, the validity of the model is examined by comparison of measured EM bias values and estimated bias values computed from a surface spectral model of the ocean. Differences in the estimated values are attributed to differences in the spectral properties between measured surfaces and model surface spectra. The largest differences are seen between measured and estimated values of the tilt modulation and surface skewness. Measured bias values are also compared with estimated bias values computed from wind and surface profile measurements of the GME data set. Estimated and measured values from the GME experiment exhibit a strong correlation over the complete data set.

In summary, the EM bias model derived here combines the contributions of previous models in a complete and simple description of the bias. Three physical mechanisms are shown to contribute to the bias, and are each related to the long wave surface statistics. The contribution of the short wave spectrum is included in the modulation coefficients, and enter the bias theory as a result of the PO scattering approximation. Using the bias theory, estimated bias values appear to more closely approximate measured bias values than previous bias models.

# Chapter 6

# **Y-ONE** Experiment

# 6.1 Introduction

The error budget for satellite-based mean sea level (MSL) measurements is dominated by the variance in EM bias estimates. Due to the variance inherent in the correlation between the EM bias and remotely measurable parameters, new methodologies are required to improve EM bias estimates. A proposed method to increase the resolution of MSL measurements is through the used of a wide swath altimeter to take off-nadir measurements of the ocean surface. A byproduct of this method is the inclusion of EM bias measurements at off-nadir incidence angles. Using the angular dependence of the EM bias, the Wide-Swath Ocean altimeter provides a means of improving EM bias estimates.

The first studies to report off-nadir measurements of the bias were by Walsh, *et al.* [65] and Hevizi, *et al.* [28]. Using airplane mounted radar instruments, the frequency dependence of the EM bias was reported at C-band and Ku-band frequencies. Due to changes in the attitude of the airplane, measurements of the EM bias were also made at off-nadir incidence angles. From these measurements, the magnitude of the EM bias was shown to decrease with incidence angle.

In this chapter data from the BYU Off-Nadir Experiment (Y-ONE) in situ study is presented. Using C-band and Ku-band altimeters off-nadir measurements of the bias were made at incidence angles from  $-3^{\circ}$  to  $17^{\circ}$ . Descriptions of the collection procedures, processing, and final data are included.
#### 6.2 Experiment Description

Measurements in the Y-ONE study were made from the Brazos A-19 natural gas platform of the Shell Exploration and Petroleum Company in the Gulf of Mexico from March 16 to April 30, 2003. The platform is located south of Houston, Texas at 28° 10′ N and 95° 35′ W with a minimum fetch of 58 km to the north. The depth of water at the Brazos A-19 platform is 40 m.

The Brazos A-19 platform complex consists of three rectangular platforms, designated B, C, and D, that are each 20 m by 50 m. Walkways of 50 m and 60 m connect platforms B and D and platforms B and C, respectively, so that the platform complex is in the shape of an L, as seen in Figure 6.1.

The Y-ONE radar and laser systems were deployed in the middle of the 60 m walkway between platforms B and C, approximately 18 m above the ocean surface. A photograph of the walkway where the instruments were deployed is shown in Figure 6.2. Ocean surface measurements were made using continuous wave Doppler radar systems with center frequencies at 5.2 and 14 GHz. Antennas for the 5.2 GHz system were dish antennas with a 60 cm diameter, corresponding to a spot size of approximately 1.8 m in diameter. The 14 GHz system employed horn antennas that were 20 cm on a side corresponding to a spot size approximately 1.7 m across. The radar systems were rotated through an angular range from 0° to 17° with measurements taken at  $-3^{\circ}$ ,  $-2^{\circ}$ ,  $-1^{\circ}$ ,  $0^{\circ}$ ,  $1^{\circ}$ ,  $2^{\circ}$ ,  $3^{\circ}$ ,  $5^{\circ}$ ,  $8^{\circ}$ ,  $11^{\circ}$ ,  $14^{\circ}$ , and  $17^{\circ}$ .

Simultaneous surface measurements were made with a system of three Optech Sentinel 3100 laser rangefinders. The rangefinders were mounted in an equilateral triangle, 1 m on a side, with one laser footprint co-located inside the footprints of both radar systems. The footprints of the laser rangefinders were approximately 13 cm in diameter, with an height measurements accurate to within 2 cm. A diagram of the configuration of the radar and laser systems in shown in the inset of Figure 6.1, and a photograph of the mounted system including the radar and laser instruments is shown in Figure 6.3.

Environmental data was collected using a Vantage Pro weather station mounted on the northwest corner of platform C at a height of 44 m. Measured wind



Figure 6.1: Diagram of the oil platform, Brazos-A19, and the experiment configuration.



Figure 6.2: View of Brazos-A19 from platform D. The 60 m walkway between platforms B and C where the radar and laser instruments are mounted is shown in the upper part of the photograph.



Figure 6.3: Deployment of the Y-ONE experiment on the walkway between Platform B and Platform C on Brazos A-19.

speeds were converted to an equivalent wind speed at 10 m by assuming a logarithmic wind velocity profile and a neutral atmosphere. This adjustment reduced measured wind speed values by approximately 15%.

## 6.3 Data Processing

Five minute records of the Y-ONE experiment include one minute samples of the local wind and weather conditions, distance measurements from each of the rangefinders at 8 Hz, and in-phase and quadrature channels of each of the radar systems sampled at 3 kHz.

#### 6.3.1 EM Bias Calculations

The EM bias is defined as the normalized correlation between the surface height of the ocean,  $\zeta$ , and the backscatter coefficient profile,  $\sigma(\zeta)$ ,

$$\epsilon = \frac{E\left[\sigma^{\circ}(\zeta)\zeta\right]}{E\left[\sigma^{\circ}(\zeta)\right]}.$$
(6.1)

Using sampled measurements of the surface profile and backscattered power, the EM bias is computed from Y-ONE measurements as

$$\epsilon = \frac{\sum_{i} \sigma_{i}^{\circ} \zeta_{i}}{\sum_{i} \sigma_{i}^{\circ}}.$$
(6.2)

To normalize the returned power measurements,  $\sigma_m^{\circ}$ , as a function of height, corrections are made to account for changes in the spot size and the free space dispersion of electromagnetic power. By combining the  $R^2$  relationship of spot size to distance, R, and the  $R^{-4}$  dependence of the backscattered power, the normalized backscatter coefficient profile,  $\sigma^{\circ}(\zeta)$ , can be expressed as

$$\sigma^{\circ}(\zeta) = \frac{K_c (R_{\circ} - \zeta)^2}{R_{\circ}^2} \sigma_m^{\circ}(\zeta)$$
(6.3)

where  $K_c$  is a calibration constant,  $R_o \approx 18$  is the mean distance to the ocean surface, and  $\zeta$  is the surface profile [12, 13].

#### 6.3.2 Data Editing

Editing of the Y-ONE data is conducted to eliminate times of instrument malfunction and spurious data values. The principle cause of instrument malfunction was traced to temporary failures of the oscillators or power supplies in the Ku-band system. These interruptions reduce the number of valid records from 11700 to a total of 9245 usable 5 minute records at C-band and 4291 records at Ku-band. From these records there are 845 and 391 nadir pointing values for the C-band and Ku-band systems, respectively.

#### 6.3.3 Power Measurements

To compute the EM bias from tower data accurate measurements of the backscattered power are required. In the Y-ONE experiment, the backscatter power measurements are computed as

$$\sigma^{\circ} = K_c \sqrt{\mathcal{I}^2 + \mathcal{Q}^2} \tag{6.4}$$

where  $\mathcal{I}$  and  $\mathcal{Q}$  refer to the in-phase and quadrature channels, respectively. The correlation between the power and surface profile measurements are then combined, as described in equation (6.1), to compute the bias.



Figure 6.4: Typical power values for a 5 minute record are plotted against surface displacement for nadir measurements of the surface. The abrupt limit on the higher power values is most likely a result of the saturation of the amplifiers in the receiver. Power levels were adjusted to restore the expected distribution in Figure 6.3.3.

During the acquisition process of the Y-ONE experiment, the backscattered power measurements of many five minute records were distorted by a nonlinearity effect. An example of the recorded power measurements can be seen in Figure 6.3.3, in black, where a cutoff for large power signals can be seen near 3 dB. This soft limit was likely caused by saturation of an amplifier in the radar instruments, where for input power levels that are too large, the signal amplification enters a non-linear region.

Because the saturation characteristics of an amplifier are one-to-one, the non-linearities introduced into the power measurements can be inverted. Inverting the saturation effects is done by fitting the average power histogram at nadir from the Y-ONE data to a similar histogram from the Gulf of Mexico (GME) data set, a previous tower experiment conducted at the same location [12]. Using a simple polynomial fit the power histogram is made to approximate the Rayleigh distribution described by Warnick, *et al.* [64]. The polynomial used to remove the saturation effects is a power series expansion about the peak,  $p_{\circ} = 0.8$ , such that

$$\sigma^{\circ} = \left\{ \begin{array}{cc} \sigma^{\circ} & p \le p_{\circ} \\ \sigma^{\circ}(1 + 0.45\delta_{p}^{3} - 0.15\delta_{p}^{4} + .04\delta_{p}^{6}) & p > p_{\circ} \end{array} \right\}$$
(6.5)

where  $\sigma^{\circ}$  is the recorded Y-ONE power measurement and  $\delta_p$  is the expansion term  $(\sigma^{\circ} - p_{\circ})$ . Power histograms for the Y-ONE data set, corrected and uncorrected, are shown in Figure 6.3.3 with a similar plot from the GME data set. Applying the correction factor, restores the linear dependence of the backscattered power on surface displacement as seen in Figure 6.3.3, where the corrected values are shown in gray.



Figure 6.5: Histograms of backscattered power values as a function of surface displacement. The clipping effect can be seen in the original Y-ONE data as an upper limit of power values. By applying the correction factor in equation (6.5), the corrected data had a distribution similar to that seen in the GME data set.

Two important points should be noted about the power correction. First, this nonlinear power scaling is displacement independent, and therefore does not add any displacement-dependent information to the data set. Rather, it makes a corrective increase to the effective weight in EM bias of high return powers, which naturally tend to correlate with negative displacements. Second, the corrupted power signals do not affect the surface profiles, so that the surface profiles used to compute the bias measurements are not changed by the power correction.

## 6.3.4 Surface Profile Verification

Direct measurements of the ocean surface displacement are made using the laser rangefinder nearest the radar systems. Due to the proximity of the laser and radar systems, the footprints of the three systems overlap when the radars were at nadir.

An oversight in data collection causes surface measurements for the laser rangefinders to begin prior to the collection of radar samples with an unknown temporal offset. This temporal offset is estimated by shifting the starting time of the laser instruments to maximize the correlation between the laser and radar surface profiles. This method ensures that the ocean surface measured by the co-located laser and radar footprints are the same.

Because Doppler radar systems do not provide absolute distance measurements, surface profiles are computed from phase angles for the radar measurements. After decimating the signal to 300 Hz, the instantaneous phase angle,  $\theta_p$  was computed as

$$\theta_p = \tan^{-1} \left( \frac{\mathcal{Q}}{\mathcal{I}} \right), \tag{6.6}$$

and unwrapped to compute the surface profile,  $\zeta$ , as

$$\zeta = \left(\frac{\theta}{2\pi}\right) \left(\frac{\lambda_{em}}{2}\right) \tag{6.7}$$

where  $\lambda_{em}$  is the electromagnetic wavelength. The covariance processing technique used by Arnold, *et al.* [12] and Melville, *et al.* [13] in previous tower experiments was also implemented and gave identical results.



Figure 6.6: Typical surface profile from C-band, Ku-band, and laser rangefinder measurements.

Previous EM bias tower experiments have noted that surface profiles computed from Doppler radar systems tend to underestimate the extrema of ocean surface profiles as a result of averaging over the radar footprint [12]. The averaging is most notable near the troughs and crests of the surface. At these times, the velocities of different parts of the footprint have different signs, so that the average velocity across the footprint is near 0, and the heights of the surface peaks and troughs are underestimated. A comparison of typical surface profiles created from the radar and laser systems can be seen in Figure 6.3.4.

Differences in the Doppler and radar measurements of the surface become more apparent when computing statistical properties of the surface, such as the significant wave height,

$$H = 4h_l, \tag{6.8}$$



Figure 6.7: Significant wave height values computed from the laser measurements are almost a constant 10 cm larger than values computed from measurements made with the C-band radar.

where  $h_l^2$  is the surface height variance. Using a Thorn infrared wave gauge and Doppler measurements of the surface profile, Arnold, *et al.* [12] show that the Doppler measurements underestimate significant wave heights in the GME data set by a constant 10 cm Arnold, *et al.* [12]. A similar relationship can be seen for Y-ONE measurements in Figure 6.3.4, where the least-squares linear fit between the significant wave height values from the two different systems is described by

$$H_{Laser} = 1.03H_{Radar} + .09, (6.9)$$

in units of meters.

To compensate for the inaccuracies inherent in the Doppler surface measurements, ocean surface statistics presented in this paper are computed from the laser rangefinder profiles. These include significant wave height, RMS wave slope, tilt modulation and surface skewness. We note that surface profiles from the laser are not used to compute bias values. Because EM bias calculations are strongly dependent on the temporal correlation of power and surface profile, the temporal shifts of the rangefinder measurements are not sufficiently accurate to compute the bias. To adjust for the height differences between the laser and radar measurements, the magnitude of the radar surface profiles are scaled according to equation 6.9 to compute the bias.

An important feature of the Y-ONE data is the presentation of another set of EM bias measurements at nadir. In support of previous data sets by Arnold, *et al.* [12] and Melville, *et al.* [13], I describe empirical relationships between the bias and sea state parameters. Previously, empirical model have been developed from tower data that describe the EM bias as a function of wind speed, significant wave height, H, and RMS wave slope, S. Other bias theories, developed from mathematical descriptions of the ocean surface, have described the bias as a function of higher order moments of the surface, such as skewness,  $\lambda_{30}$ , and tilt modulation,  $\lambda_{12}$  [5, 16, 27]

#### 6.3.5 Long Wave Parameter Definitions

Values of the RMS wave slope are computed by discretizing the statistical definition of the RMS slope,

$$S^2 = \int k^2 W(k) dk, \qquad (6.10)$$

where W(k) is the long wave surface PSD [13, 64]. By making use of the deep wave dispersion relation,

$$k = \frac{\omega^2}{g},\tag{6.11}$$

the RMS slope can be computed using the relation between the temporal PSD,  $W_T(\omega)$ , and the spatial PSD, W(k),

 $\int \left(\frac{\omega^2}{g}\right)^2 W_T(\omega) d\omega = \int k^2 W(k) dk, \qquad (6.12)$ 

developed by [34].

The long wave skewness parameter is defined as the third central moment of the surface profile normalized by the cube of the standard deviation, and can be computed as

$$\lambda_{30} = \frac{E[\zeta^3]}{E[\zeta]^{3/2}}.$$
(6.13)



Figure 6.8: The relationship between the EM bias values and significant wave height from the Y-ONE data set.

where  $\zeta$  is the long wave surface profile.

The long wave tilt modulation combines measurements of the surface displacement and slope as a measure of the correlation between wave height and surface tilt. Defined as,

$$\lambda_{12} = \frac{E\left[\zeta\zeta_x^2\right]}{E\left[\zeta\right]^{1/2}E\left[\zeta_x\right]},\tag{6.14}$$

the tilt modulation is computed using principles of the weakly nonlinear theory, so that

$$\lambda_{12} = \frac{4}{h_l s_l^2} \int_{k_{sep}}^{\infty} dk W(k) \int_0^k dl \left(2k^2 l + l^3\right) W(l), \tag{6.15}$$

as described by Jackson [15].

## 6.3.6 EM Bias at Nadir

Values of the EM bias ranged from -10.4 cm to 0.5 cm for the C-band system and from -8.0 cm to 0.0 cm at Ku-band over the course of the Y-ONE



Figure 6.9: Correlation of the RMS wave slope, S, with the relative bias,  $\beta$ , at C-band (+) and Ku-band (o).

experiment. When plotted against significant wave height, in Figure 6.3.5, the first order, linear relationship with the significant wave height is clearly seen. A least squares fit of the EM bias to the significant wave height for Y-ONE is

$$\epsilon_C(cm) = -3.38H + 1.46\tag{6.16}$$

$$\epsilon_{Ku}(cm) = -3.04H + 0.7 \tag{6.17}$$

where the subscripts C and Ku refer to the frequency of the scatterometer. Values of the normalized bias, defined as

$$\beta = \frac{\epsilon}{H},\tag{6.18}$$

range from -4.3% to 0.7% of the H at 5 GHz and from -3.53% to -0.1% of H at 14 GHz.

Empirical relationships between the RMS slope and relative bias have been investigated in a number of *in situ* and laboratory experiments [13, 23, 64, 66]. Figure 6.3.6 shows the relationship between  $\beta$  and S from the Y-ONE data where the



Figure 6.10: Correlation of the tilt modulation,  $\lambda_{12}$ , with the relative bias,  $\beta$  at C-band (+) and Ku-band (o).

linear correlation reported in previous experiments can be clearly seen. Values of the RMS slope range from .04 to .14 over the course of the experiment.

The relationship of the relative bias with the long wave skewness and tilt modulation can be seen in Figure 6.3.6 and Figure 6.3.6, where a negative correlation between the  $\lambda_{12}$  and  $\lambda_{30}$  and  $\beta$  is consistent with the bias theory in equation (5.45). I also note that the relative bias is less correlated with the skewness than with S and  $\lambda_{12}$ . The tilt modulation ranges from 0.07 to 0.39, and skewness varies from -0.6 to 0.7.

#### 6.3.7 GME and Y-ONE Comparison

The GME and Y-ONE data sets were collected from the same location, allowing for a comparison of EM bias and surface statistics. With the Y-ONE experiment deployed during March and April, 2003, and the GME experiment deployed



Figure 6.11: Correlation of the surface skewness,  $\lambda_{30}$ , with the relative bias,  $\beta$  at C-band (+) and Ku-band (o).

from December to May, 1991-1992, the data also had an overlap with respect to the time of year.

Because the profile statistics from the Y-ONE data are computed using the laser rangefinders, they are not directly comparable to the GME data. To make a direct comparison between bias measurements from the two experiments, GME values of the significant wave height and RMS slope must be adjusted. For the significant wave height, the relationship between laser and radar data is addressed in equation (6.9). Using Y-ONE data, radar and laser measurements of the RMS slope can be related by

$$S_{Laser} = 1.11S_{Radar} + 0.014. (6.19)$$

A plot of the relationship is shown in Figure 6.3.7.

Using these relationships to modify the GME values of S and H allows a direct comparison of the bias as a function of the significant wave height, seen in



Figure 6.12: Comparison of RMS slope values computed from the laser and C-band radar measurements.

Figure 6.3.7. The relationship of  $\epsilon$  and H has a similar magnitude and slope in both data sets.

The relationship of the relative bias with RMS slope for both data sets is shown in Figure 6.3.7, using the correction factor from equation (6.19).

#### 6.4 Summary

This chapter presents off-nadir EM bias measurements from the BYU Off-Nadir Experiment (Y-ONE) conducted in the Gulf of Mexico in 2003. Using C-band and Ku-band radar systems and laser altimeters, the EM bias is calculated directly from ocean surface profiles and backscattered energy returns. A description of the processing methods and modeling parameters is followed by a summary of the Y-ONE data set. Y-ONE measurements are compared with data from a previous experiment conducted from the same platform. The comparison shows similar values for the bias



Figure 6.13: First order dependence of the EM bias on significant wave height for the GME and Y-ONE data sets.



Figure 6.14: Relationship of the relative bias and RMS wave slope for the GME and Y-ONE data sets. The same linear dependence can be seen in both data sets.

and surface statistics from the two data sets, including the correlation of the bias with significant wave height and RMS slope.

Using measurements from the Y-ONE experiment, the angular dependence of the bias is shown to be caused by a phase shift between the surface profile and backscattered power. The resulting phase shift causes the magnitude of the bias to decrease at small incidence angles.

Estimated bias values calculated using the off-nadir bias theory show good agreement with measurements from the Y-ONE data set. The magnitude of estimated and measured values of the bias are shown to decrease with incidence angles up to approximately,  $\theta = 15^{\circ}$ , and have increasing positive values for larger incidence angles. An empirical fit of  $\beta(\theta) = \beta(0) \cos(\theta \pi/\theta_{\circ} 2)$  is developed from the measured Y-ONE bias values.

## Chapter 7

# **Off-Nadir EM Bias Model**

### 7.1 Introduction

In this chapter the nadir EM bias theory developed in Chapter 5 is generalized to off-nadir incidence angles. The nadir EM bias theory is derived using the physical optics (PO) scattering approximation, and includes the effects of hydrodynamic modulation and non-Gaussian long wave statistics. In generalizing to off-nadir incidence angles, the EM bias is shown to have an angular dependence that is a sum of the hydrodynamic and tilt modulation biases. The resulting theory shows that the magnitude of the EM bias decreases with incidence angle.

#### 7.2 Off-Nadir Bias Model

Generalizing the EM bias model in equation (5.45) to off-nadir incidence angles is accomplished by including angular dependent terms in the PO scattering model, so that the off-nadir bias theory reflects changes in the short wave scattering with angle.

#### 7.2.1 Derivation

From the definition in equation (6.1), the EM bias can be rewritten as,

$$\epsilon = \frac{\int \int \zeta \sigma^{\circ}(\zeta, \theta) \mathbf{P}(\zeta, \theta) \mathrm{d}\zeta \mathrm{d}\theta}{\int \sigma_{\circ}(\zeta, \theta) \mathbf{P}(\zeta, \theta) \mathrm{d}\zeta \mathrm{d}\theta},\tag{7.1}$$

where  $P(\zeta, \theta)$  is the joint height-slope distribution determined by the long ocean waves. This formulation describes the bias as scattering from small wave surface facets where the local incidence angles and heights are described by  $P(\zeta, \theta)$ .

By assuming small local incidence angles such that

$$\theta \approx \tan \theta \approx \zeta_x,\tag{7.2}$$

and using the weakly non-linear theory [14] the joint-height distribution of the long waves can be described as a Gram-Charlier expansion of a Gaussian distribution [5, 16],

$$P(\zeta, \zeta_x) = \frac{e^{-\frac{1}{2}(\eta^2 + \eta_x^2)}}{2\pi\sigma\sigma_x} \times \left[1 + \frac{\lambda_{30}}{6}\mathcal{H}_{30}(\eta, \eta_x) + \frac{\lambda_{12}}{2}\mathcal{H}_{12}(\eta, \eta_x)\right].$$
(7.3)

To simplify notation, the PDF is expressed in terms of the normalized height,  $\eta = \zeta/h_l$ , and the normalized surface slope,  $\eta_x = \zeta_x/s_l$ , where  $h_l^2$  and  $s_l^2$  are surface height variance and surface slope variance, respectively. The symbols  $\mathcal{H}_{30}(\eta, \eta_x)$  and  $\mathcal{H}_{12}(\eta, \eta_x)$  refer to Hermite polynomials defined by

$$\mathcal{H}_{30}(\eta,\eta_x) = \eta^3 - \eta \tag{7.4}$$

$$\mathcal{H}_{12}(\eta, \eta_x) = \eta(\eta_x^2 - 1).$$
 (7.5)

Scattering by the short ocean waves is modeled using the physical optics scattering approximation

$$\sigma^{\circ}(\psi) = \frac{k_{em}^2 \cos^2 \psi}{4\pi} \int \int e^{ik_b x} e^{-\lambda(1 - C(x,y))} dx dy$$
(7.6)

where  $\psi$  is the incidence angle of the electromagnetic wave,  $k_b = 2k_{em} \sin \psi$  and  $\lambda = (2k_{em}h_s \cos \psi)^2$ . Note that for  $\psi = 0$ , the terms  $k_b = 0$  and  $\cos(\psi) = 1$  which is the result used to create the EM bias model at nadir. The correlation function

$$C(x,y) = \frac{1}{h_s^2} \int W(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{k}$$
(7.7)

is the Fourier transform of the isotropic, normalized short wave PSD,  $h_s^{-2}W(\mathbf{k})$  where  $h_s^2$  is the small wave height variance. For the remainder of the paper, the explicit dependencies of the correlation function are dropped so that C = C(x, y).

To include hydrodynamic modulation in the nadir EM bias model, we approximate the short wave modulation described by the modulation transfer function of Alpers and Hasselmann [63] as a linear function of the surface displacement,

$$h_s = h_\circ \left(1 + m\eta\right),\tag{7.8}$$

where  $h_{\circ}$  is the average short wave height, and m is the modulation coefficient. As part of the analysis of m in Chapter 5, it is shown that the relationship between the modulation coefficient and RMS wave slope is nearly linear so that  $m = \nu S$ . By using this relationship and substituting equation (7.8) into the PO scattering model, equation (7.6), the backscatter profile includes the hydrodynamic modulation in the EM bias model as a function of the RMS slope.

The final form of the EM bias model is created by substituting the joint height slope PDF from equation (7.3) and the PO approximation from equation (7.6), into the EM bias definition, (7.1) and integrating out the slope dependence so that

$$\epsilon = -H\left(\gamma\nu S + \tau\lambda_{12} + \kappa\lambda_{30}\right). \tag{7.9}$$

This form is identical to the nadir EM bias model in equation (5.45), with the angular dependence of the model contained in the small wave coefficients

$$\gamma = \frac{1}{2} \frac{\int \int \lambda_{\circ}(1-C) e^{i2k_{em}x\psi} e^{-\lambda_{\circ}(1-C)} e^{-\mu^{2}/2} dx dy}{\int \int e^{-\lambda_{\circ}(1-C)} e^{-\mu^{2}/2} dx dy}$$
(7.10)

$$\tau = \frac{1}{8} \frac{\int \int e^{i2k_{em}x\psi} \mu^2 e^{-\lambda_0(1-C)} e^{-\mu^2/2} dx dy}{\int \int e^{-\lambda_0(1-C)} e^{-\mu^2/2} dx dy}$$
(7.11)

$$\kappa = \frac{1}{24},\tag{7.12}$$

where  $\mu = 2xk_{em}s_l, \lambda_{\circ} = (2k_{em}h_{\circ})^2$ .

## 7.2.2 Model Analysis

In equation (7.9), the EM bias is described as a function of long wave surface statistics, S,  $\lambda_{12}$ , and  $\lambda_{30}$  modified by the small wave coefficients,  $\gamma$ ,  $\tau$ , and  $\kappa$ . More accurately, the small wave coefficients are functions of EM scattering from the small ocean waves. This dependence on EM scattering causes the small wave coefficients  $\gamma$  and  $\tau$  to be the source of the angular dependence in the off-nadir EM bias model.

The small wave coefficients are modeled using a power law PSD computing numerical values for  $\gamma$  and  $\tau$  requires the specification of the exponent p, the separation wavenumber,  $k_{sep}$ , and an average small wave surface height variance,  $h_{\circ}^2$ . The



Figure 7.1: Dependence of the hydrodynamic modulation coefficient,  $\gamma$ , with incidence angle for k = 100 and minimum wavenumber of  $k_{min} = 2\pi$ . For a separation wave number of  $k_{sep} = 2\pi$ , typical values of  $h_{\circ}$  for an ocean surface are greater than .01 m.

value of the separation wavenumber is fixed at  $k_{sep} = 4\pi/5$ , and I refer the reader to Chapter 5 for further discussion of the impact of  $k_{sep}$  on the estimated EM bias values. The exponent of the small wave power spectrum is also approximated as p = 3. With these values, the incidence angle dependence of  $\gamma$  is shown in Figure 7.2.2 for various values of  $h_{\circ}$ . For the same short wave PSD values, the angular dependence of  $\tau$  is shown in Figure 7.2.2.

The angular dependence of  $\gamma$ , caused by differences in roughness at the crests and troughs of the waves, creates the hydrodynamic bias. At large incidence angles the more Lambertian scattering from the wave crests will reverse the bias effect so that there is a larger return from the crests than from the troughs. At small incidence angles, shown in Figure 7.2.2, this transition from positive to negative values can be seen.



Figure 7.2: Dependence of the tilt modulation coefficient,  $\tau$ , with incidence angle for  $k_{em} = 100$ . For a separation wave number of  $k_{sep} = 2\pi$ , typical values of  $h_{\circ}$  for an ocean surface are greater than .01 m.

Changes in the tilt modulation bias with angle are caused by a larger concentration of horizontal scattering facets near the troughs than the crests of ocean waves. This results in a larger specular return for nadir pointing instruments. With increasing incidence angle, the steeper crests create a larger backscatter return than the flatter troughs, and the bias effect is reversed. It is also of interest to note that with larger small wave heights the angular dependence of the tilt modulation is greatly reduced. This effect results from a more Lambertian EM scattering pattern for larger small waves. For large enough waves this EM scattering becomes isotropic, and the tilt modulation bias vanishes.

Different from  $\gamma$  and  $\tau$ , the value of  $\kappa$  in the off-nadir bias model does not show an angular dependence. In a study on bistatic scattering from an ocean surface with skewness, Picardi, *et al.* [67] showed that the backscattered power has a angular dependence described by  $\cos \psi$ . With the small incidence angle approximation used in this study, this effect can be neglected so that the skewness coefficient does not have an angular dependence.

#### 7.3 Off-Nadir Measurements

In this section I describe the angular dependence of the EM bias as seen in the Y-ONE data set and the off-nadir bias model. As part of this discussion I analyze the cause of the off-nadir dependence in the Y-ONE measurements. A simple equation that can be used to describe the off-nadir dependence of the bias is also developed.

#### 7.3.1 Cause

From equation (7.1), the bias is caused by the negative correlation between the backscattered power and surface displacement. This relationship is shown in the upper axis in Figure 7.3.1, where the cross-correlation function between  $\sigma^{\circ}$  and  $\zeta$ is shown. At off-nadir incidence angles, a phase shift between  $\sigma^{\circ}$  and  $\zeta$  is introduced. This shift can be seen in the lower axis of Figure 7.3.1, where an obvious time shift can be seen in the maximum correlation between  $\sigma^{\circ}$  and  $\zeta$ . The result is an effective decorrelation that reduces the magnitude of the bias for small off-nadir



Figure 7.3: Correlation of backscattered power and surface height at  $0^{\circ}$  and  $17^{\circ}$  incidence angles. The temporal shift in the minimum correlation point causes a decrease in the magnitude of the EM bias values.

incidence angles. With larger incidence angles, the phase shift becomes larger, until the backscattered power and surface displacement are in phase, resulting in a positive correlation and positive bias values.

#### 7.3.2 Theory Estimates

In the development of the off-nadir bias theory, the angular dependence is shown to be a function of the small wave height,  $h_s$ , through the bias coefficients,  $\gamma$  and  $\tau$ . Examples of the angular dependence of the bias are shown in Figure 7.3.4 for different values of S and  $\lambda_{12}$ . The bias coefficients,  $\gamma$  and  $\tau$ , are computed by approximating the short wave height as a constant  $h_s = .02$  m. Using this constant value and setting  $\nu = .6$ , as a typical value from general ocean surface conditions. Because  $\gamma$  and  $\tau$  are dependent on the small wave surface height, the constant value of  $h_s$  leads to example curves with the same incidence angle for  $\beta = 0$ . This effect can be seen in Figure 7.3.4 where example curves show  $\beta = 0$  near  $\theta = 16^{\circ}$ . For different small wave conditions the intersect angle can change such that larger values of  $h_s$  result in a larger intersect angle.

#### 7.3.3 Y-ONE Measurements

Average relative bias measurements as a function of angle are shown in Figure 7.3.4 with error bars indicating one standard deviation of the data. As anticipated, measured bias values increase from a minimum value at  $\theta = 0$ , to  $\beta = 0$  near 15°. For incidence angle larger than 15°, the average bias value is positive.

#### 7.3.4 Empirical Fit

To simplify the relationship of the bias with incidence angle, a fit to the model can be developed from the Y-ONE measurements. Using the value of the bias at nadir,  $\beta(0)$ , and the angle,  $\theta_{\circ}$ , at which  $\beta = 0$  a cosine curve is fit to the average bias values so that,

$$\beta(\theta) = \beta(0) \cos\left(\frac{\pi}{2}\frac{\theta}{\theta_{\circ}}\right).$$
(7.13)

With average values from the Y-ONE data of  $\beta(0) = -2.58$  and  $\theta_{\circ} = 15^{\circ}$ , an empirical fit to the average Y-ONE data is shown in Figure 7.3.4. The result is an empirical fit to the data that requires only the normalized bias at zero and the estimated zero-bias intersect angle.

#### 7.4 Summary

To estimate the angular dependence of the bias, an off-nadir bias model is developed by generalizing a recently developed PO-based EM bias model to offnadir incidence angles. The resulting model describes the bias as function of long wave surface parameters, with coefficients that are functions of EM scattering from short ocean waves. Changes in the distribution of backscattered power with incidence angle are reflected in the hydrodynamic modulation coefficient and tilt modulation coefficient, leading to a decrease in magnitude of the bias as a function of incidence angle.



Figure 7.4: Measured and estimated relative bias values at off-nadir incidence angles. Values of the RMS slope, S, and tilt modulation,  $\lambda_{12}$ , are shown in the legend. Bars of one standard deviation indicate the extent of the measured EM bias values from the Y-ONE data set. An empirical fit to the data described by  $\beta(\theta) = \beta(0) \cos(\theta \pi/\theta_0 2)$  is also shown, where  $\theta_0$  is estimated at 15°.

# Chapter 8

## Conclusion

In July, 1991 scientists gave the first public predictions of the upcoming  $El Ni\tilde{n}o$  event. With the months of warning, governments and relief agencies were able to implement plans to cope with the incipient flooding, hurricanes, and drought that accompany the  $El Ni\tilde{n}o$  event. These preparations are estimated to have saved hundreds of lives and millions of dollars worth of damage [68].

Advanced preparation for El Niño is one of many results of an improved understanding of the forces of nature. By increasing our knowledge of the geophysical forces of the planet we have also created better local weather prediction, safer shipping channels, and improved fishing yields. These, and many other, advances create a better life for the human race.

## 8.1 Contributions

The research contributions in this dissertation are part of the effort to improve our measurements of the mean sea level. More specifically, I focus on improving EM bias estimates. As the largest source of error in altimeter measurements, improvements in this area can lead to better understanding in other areas. Specific contributions in this dissertation include improved empirical modeling, the development of a theoretical bias model, and an *in situ* EM bias experiment.

#### 8.1.1 Model Analysis

Empirical EM bias models are limited by the correlation of the bias with the estimation parameters. For operational models, the parameters that are most commonly used are the wind speed and significant wave height. By analyzing bias models created from these parameters I show that the accuracy of current operational bias models is limited by the inherent variability between the estimation parameters and the bias.

With these inherent limitations, the accuracy of empirical models can only be improved by using other estimation parameters. The estimation parameter used in this study is the RMS wave slope. A study of the dependence of the relative EM bias as a function of the slope, wind speed, and significant wave height, shows that the EM bias is most strongly correlated with the RMS wave slope. In addition, the information in the significant wave height is complementary to the RMS wave slope so that by combining the two parameters, the variance in EM bias estimates is reduced to less than 1 cm.

#### 8.1.2 Nonparametric Regression

Efforts to improve operational EM bias models also include modeling schemes beyond the normal least squares estimates. One modeling technique known as nonparametric regression (NPR) creates local polynomial estimates of the bias, rather than a global fit from the entire data set. The result is a model that can more accurately model the bias value in data poor regions of the of the parameter space, reducing the overall estimation errors.

In applying nonparametric modeling techniques to the GME data set, the resulting bias models shows slight improvement over models created using the least squares method. Of more importance is the increased similarities between the satellite and tower based models. In using the nonparametric estimation techniques the historical offset between the models is significantly reduced. I also show that using NPR in conjunction with the RMS wave slope creates models with an error variance near 0.33 cm.

#### 8.1.3 Rough Surface Scattering

The next area of research is a study of the validity of rough surface scattering models. Of primary interest is the physical optics (PO) scattering approximation, and its application to surfaces described by power law and Gaussian power spectral densities. Using numerical simulations of electromagnetic scattering from a one-dimensional surfaces, the validity and angular dependence of the scattering models is determined. Results from the study show that the classic PO validity criterion, based on surface curvature, does not accurately describe the region of validity. Instead, a new validity criterion is developed that is a function of the surface slope. Validity criteria for the other PO-based models are also developed, and show a similar dependence on the surface slope parameter.

## 8.1.4 EM Bias Model

A central contribution of this dissertation is the development of an EM bias theory using the PO approximation. By combining hydrodynamic modulation, tilt modulation, and skewness into a single bias theory, a comprehensive EM bias model is developed. The bias theory includes contributions from previous bias models in a form that is easier to compute, and more descriptive of the physical mechanisms that create the bias. A comparison of estimated bias values with *in situ* measurements also shows that the bias theory is more representative of measured bias values.

A significant development in this model is the inclusion of the PO approximation to model the bias. Because most previous models have been developed using the simpler GO approximation, the EM scattering from centimeter scale waves is neglected. In the analysis of the bias theory, I show that scattering from the small scales waves has a major impact on the final bias value. This understanding of the physical causes of the bias is an important contribution to the EM bias community.

#### 8.1.5 BYU Off-Nadir Experiment

The design, deployment, and analysis of the Brigham Young University Off-Nadir Experiment (Y-ONE) resulted in three important contributions to the study of the EM bias. First, the Y-ONE study provided another set of *in situ* measurements of the EM bias in continuation of the GME data set. Second, the deployment of the laser rangefinders confirm measurements of the Doppler radar systems. The final result is the first extensive study of the angular dependence of the EM bias. Using measurements from C-band and Ku-band radar systems, measurements of the EM bias are taken at incidence angles from  $0^{\circ}$  to  $17^{\circ}$ . These measurements show that the magnitude of EM bias values decrease with incidence angle up to approximately  $17^{\circ}$  where the bias approaches 0. This experiment is conducted in support of the recently proposed Wide Swath Ocean Altimeter, that will use the angular dependence of backscatter measurements to improve estimates of the mean sea level.

#### 8.1.6 Off-Nadir EM Bias Model

The final contribution is the generalization of the EM bias model to offnadir incidence angles. Changes in the EM bias with angle are caused by the angular dependence of the short wave scattering. Because of the inclusion of the PO approximation, the importance of the short wave scattering is better understood, and the physics that cause the bias are more clearly described. Off-nadir estimates of the bias are also shown to accurately model the angular dependence of the measured EM bias values from the Y-ONE experiment.

#### 8.2 Future Research

The contributions in this dissertation have advanced the understanding of the EM bias in a number of ways. With these advances come the possibility of other lines of research that can be further investigated.

#### 8.2.1 Two-dimensional Model

The derivation of the bias model described in this dissertation uses the assumption of long crested, long waves. This assumption explicitly reduces the long wave surface spectrum to a one-dimensional surface. A further area of research could use a more complete description of the EM bias that includes two-dimensional long wave surface statistics. With the two-dimensional model the model will include a complete description of the ocean surface.

## 8.2.2 Angular Dependence

Due to the nature of satellite measurements, theoretical models derived from the surface statistics are not directly applicable. Theoretical models describe the bias as function of long wave surface statistics with scattering from the short wave facets, but large footprints of the satellite based measurements preclude the measurement of the small scale surface features.

The development of the Wide-Swath Ocean Altimeter, has been proposed as a method to improve the accuracy of mean sea level measurements. By using multiple look angles, the angular dependence of the backscattered energy can be exploited to improve the estimates. Included in the changes of the backscattered energy are changes in the bias. With the multiple look angles, the changes in bias measurements can be fit to a curve described by the off-nadir bias model, yielding a more accurate estimate of the bias at nadir.

## 8.2.3 Wave Slope Measurements

The dependence of the EM bias on the RMS wave slope has been well established in this research. However, the measurements that have been reported from the tower experiments are based on a number of assumptions relating surface profiles in time to spatial slope values. A check of the validity of the RMS slope approximation used in this work would be helpful by including spatial measurements of the ocean surface. The laser rangefinders used in the validation studies of the radar data can be deployed and more accurately calibrated to attain sufficient accuracy for these measurements.

## 8.2.4 Tilt Modulation

A salient feature of the EM bias model is the dependence of the bias on two long wave statistics, RMS wave slope and tilt modulation. Due to data from *in situ* experiments, laboratory measurements, and theoretical studies, knowledge on the RMS wave slope and its relationship with the bias is fairly extensive. Conversely, little is known about the tilt modulation. Questions remain about the physical mechanisms that cause the bias, its relationship with the long and short wave surface spectra, and its correlation with the RMS slope. Further studies on the tilt modulation and its relationship with the EM bias could provide significant information to improve the accuracy of EM bias estimates.

## 8.3 Summary

With the contributions made in this dissertation, understanding of the EM bias has been greatly improved. By including contributions that relate to current operational models, the development of a more accurate EM bias theory, and the ground work for future methodologies I have touched on many of the major issues involved in this area of research.

APPENDICES
## Appendix A

## Definition of the Electromagnetic Bias

The electromagnetic bias results from a difference between the true mean sea level (MSL),  $\zeta_{\circ}$ , and the surface height associated with the median backscattered power measured by the altimeter. By setting the true MSL to zero, the EM bias can then be expressed as

$$\epsilon = \zeta_{\circ} - \hat{\zeta}_{\circ} \tag{A.1}$$

$$= -\hat{\zeta}_{\circ}. \tag{A.2}$$

From [15], it is noted that for satellite based altimeter applications, the impulse response, p(x), of the surface is approximately proportional to the scattering cross-section density,  $\sigma^{\circ}(z)$ , such that

$$p(x) = \int_{-c\tau/2}^{x} \sigma^{\circ}(z) \mathrm{d}z \tag{A.3}$$

where c is the speed of light, and  $\tau$  is the time delay between the MSL and the highest surface peak. With this definition, the approximated MSL,  $\hat{\zeta}_{\circ}$ , can be computed as the distance when

$$\frac{\int_{-\infty}^{x} \sigma^{\circ}(z) \mathrm{d}z}{\int_{-\infty}^{\infty} \sigma^{\circ}(z) \mathrm{d}z} = \frac{1}{2}.$$
(A.4)

With the assumption of an almost Gaussian distribution of  $\sigma^{\circ}(z)$ , the lower limit on the integrals can been changed to  $\infty$  with little effect on the result.

Interpreting equation A.4 as a PDF, the value of  $\hat{\zeta}_{\circ}$ , is seen to be the median of the returned power. By approximating the median with the mean for the

near-Gaussian,  $\sigma^{\circ}(z)$ , the EM bias can be expressed as

$$\epsilon = \int z \left( \frac{\sigma^{\circ}(z) dz}{\int \sigma^{\circ}(z) dz} \right).$$
(A.5)

$$= \frac{\int z\sigma^{\circ}(z)\mathrm{d}z}{\int \sigma^{\circ}(z)\mathrm{d}z}.$$
 (A.6)

Expressing this in terms of the expected value results in the common expression of the EM bias,

$$\epsilon = \frac{E[z\sigma^{\circ}(z)]}{E[\sigma^{\circ}(z)]} \tag{A.7}$$

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