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Compositing Slice Kp

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Abstract

High resolution wind retrieval involves using higher resolution measurements known as "slices" to retrieve the wind. These slice measurements are formed by using range and doppler discrimination on the whole pulse measurement known as "egg" measurements. At the present time, L2B wind retrieval is done using only the egg measurement. However, compositing slices can yield higher resolution winds. We found that to achieve accurate wind retrieval, it is essential to correctly compute the composited σ^0 K_p coefficients. In this paper the method for compositing the K_p coefficients is derived.

1 Introduction

Wind retrieval requires the knowledge of the variance of each measurement made by the scatterometer. The Seawinds instrument is capable of making two types of measurements. The first type, refered to as an egg, is made from the total power returned from the area of the footprint. The second type, refered to as a slice, is formed by using range and doppler discrimination to get the power returned from a smaller section or slice of the footprint. These slices are used in the process of retrieving wind at a higher resolution.

The Seawinds data reports the predicted variances of both the egg measurement and the slice mesurements. For composite wind retrieval slices are averaged together to form a composite measurement which is then used in the wind retrieval process. The variance of these composite measurements is needed to acurately retrieve the wind estimate. It is therefore of interest to derive a method to calculate the variance of the composite measurements from the slice measurement variance. To do this we derive a method to average the slice measurement variances to get that of the egg measurement.

2 K_p

The accuracy of wind measurement made using a scatterometer depends greatly on the accuracy of the σ^o measurements taken by the scatterometer. A commonly used metric for measuring the accuracy of the σ^o measurement is K_p, sometimes referred to as K_{pc}. K_p is defined to be the normalized standard deviation of the σ^o measurement given by

$$K_p = \frac{\sqrt{VAR[\sigma^o]}}{E[\sigma^o]}, \tag{1}$$

where $VAR[\sigma^o]$ is the variance of the σ^o measurements and $E[\sigma^o]$ is the mean of the σ^o measurements. For the Seawinds instrument aboard the QuikSCAT satellite, K_p for egg measurements is given by

$$K_p^2 = A^e + \frac{B^e}{SNR} + \frac{C^e}{SNR^2} \tag{2}$$

where SNR is the signal to noise ratio (P_s^e/P_n), A^e, B^e, and C^e are given (approximately) by

$$A^e = \frac{1}{B_{3dB}T_p} \tag{3}$$

$$B^e = \frac{2}{B_{egg}T_g} \tag{4}$$

$$C^e = \frac{1}{B_{egg}T_g}\left(1 + \frac{B_{egg}}{B_n}\right), \tag{5}$$

B_{egg} is the total egg bandwidth, T_p is the pulse length, T_g is the range gate length, B_{3dB} is the 3dB bandwidth of the total dechirped echo return, and B_n is the noise channel bandwidth.

3 K_p for Slices

K_p reported for slices in the QSCAT L1B data product has the same form as the egg K_p (Eq. 2). For slices A^s, B^s, and C^s are given by,

$$A_i^s = \frac{1}{B_sT_p} \tag{6}$$

$$B_i^s = \frac{2}{B_sT_g} \tag{7}$$

$$C_i^s = \frac{1}{B_sT_g}\left(1 + \frac{B_s}{B_n}\right) \tag{8}$$

where, as with the egg K_p coefficients, T_p is the pulse length, T_g is the range gate length, B_n is the noise bandwidth and B_s is the slice bandwidth. However, the approximation represented by these equations is not as good as for the egg.

Comparison of the egg and slice K_p coefficients reveals that they are essentially equal with the exception of the bandwidth of the measurement. In the egg K_p case B_{egg} and B_{3dB} are used. In [1] the total egg bandwidth, B_{egg}, is defined to be the sum of the bandwidths of the slices which are contained in the egg. B_{3dB} is a function of scan azimuth angle and orbit position. Therefore A^e becomes a function of azimuth angle and orbit position while A^s is treated as a constant for each pulse. A precise formulation is beyond the scope of this report, but we will seek a good approximation.

4 Compositing Slice K_p

To derive a method to composite K_p, an understanding of the normalized radar cross-section σ^o is given. σ, the unnormalized radar cross-section is a function of the effective area of the scatterer, the amount of energy absorbed by the scatterer and the gain of the scatterer in the direction of the receiver. This is given by [2]

$$\sigma = A_{rs}(1 - f_a)G_{ts}. \tag{9}$$

Here f_a is the fraction of power absorbed by the scatter, A_{rs} is effective area of the scatter, G_{ts} is the gain of the scatterer in the direction of the receiver. The normalized radar scattering cross-section is given by

$$\sigma^o = \frac{\sigma}{A_{rs}}. \tag{10}$$

The total power measured at the receive antenna is then given by

$$P_r = \frac{P_t G^2 \lambda^2 \sigma^o}{(4\pi)^3 R^4 A_{rs}} \tag{11}$$

where P_t is the transmitted power, G is the gain of the antenna, λ is the wave length of the transmitted power, and R is the slant range from the antenna to the scatterer. This equation is known as the radar equation [2]. It can be written simply as

$$P_r = X\sigma^o \tag{12}$$

where X is the lumped elements of the radar equation. X is known as the X_{factor} in the L1B data.

4.1 Compositing σ^o

One of the first considerations before derivating a method for compositing K_p, is to derive the compositing method for σ^o.

The total power measured at the antenna is the sum of the power from each slice, i.e.

$$P_e = \sum_i P_{s,i}. \tag{13}$$

The total power is the power in the egg measurement (P_e). Substitution of Eq. 12 into Eq. 13 gives

$$\sigma^o_e X_e = \sum_i \sigma^o_{s,i} X_{s,i}. \tag{14}$$

The X_{factor} for the egg, X_e is also the sum of each slice X_{factor} for a given pulse

$$X_e = \sum_i X_{s,i}. \tag{15}$$

Therefore σ^o_e can be expressed as the weighted average of $\sigma^o_{s,i}$ for each slice i in the given composite

$$\sigma^o_e = \frac{\sum\limits_{i=1}^{n} \sigma^o_{s,i} X_{s,i}}{\sum\limits_{i=1}^{n} X_{s,i}}. \tag{16}$$

4.2 Compositing K_p

To composite the slice K_p recall that K_p is defined to be the normalized standard deviation of the σ^o measurement,

$$K_p = \frac{\sqrt{Var[\sigma^o]}}{E[\sigma^o]}. \tag{17}$$

To derive a compositing method for K_p the variance and mean of σ^o must be examined. The variance of σ_e^o is given by

$$\begin{aligned}
Var[\sigma_e^o] &= E[(\sigma_e^o)^2] - E[\sigma_e^o]^2 \\
&= E\left[\left(\frac{P_e}{X_e}\right)^2\right] - E^2\left[\frac{P_e}{X_e}\right] \\
&= E\left[\left(\frac{\sum_i P_{s,i}}{\sum_i X_{s,i}}\right)^2\right] - E^2\left[\frac{\sum_i P_{s,i}}{\sum_i X_{s,i}}\right] \\
&= \frac{\sum_i \sum_j E\left[P_{s,i}P_{s,j}\right]}{\left(\sum_i X_{s,i}\right)^2} - \left(\frac{\sum_i E[P_{s,i}]}{\sum_i X_{s,i}}\right)^2.
\end{aligned} \tag{18}$$

A simple noise model is [3]

$$\sigma_{s,i}^o = \sigma_s^o(1 + \nu_i K_{p,i}) \tag{19}$$

where ν_i is a unit variance, zero mean random variable which is uncorrelated from slice to slice. σ_s^o is assumed to be a constant. Substituting Eq. 12 into Eq. 19 gives

$$\begin{aligned}
\frac{P_{s,i}}{X_{s,i}} &= \sigma_s^o(1 + \nu_i K_{p,i}) \\
P_{s,i} &= \sigma_s^o X_{s,i}(1 + \nu_i K_{p,i}).
\end{aligned} \tag{20}$$

With this model the cross correlation of the power in each slice is given by

$$\begin{aligned}
E\left[P_{s,i}P_{s,j}\right] &= E[(\sigma_s^o)^2 X_{s,i}X_{s,j}(1 + \nu_i K_{p,i})(1 + \nu_j K_{p,j})] \\
&= (\sigma_s^o)^2 X_{s,i}X_{s,j}(1 + K_{p,i}K_{p,j}\delta_{i,j}).
\end{aligned} \tag{21}$$

The expected value of the power in the i^{th} slice can be written as

$$\begin{aligned}
E[P_{s,i}] &= \sigma_s^o X_{s,i}(1 + K_{p,i}E\nu_i) \\
&= \sigma_s^o X_{s,i}.
\end{aligned} \tag{22}$$

The variance and mean can now be written as

$$Var[\sigma_e^o] = \frac{\sum_i \sum_j (\sigma_s^o)^2 X_{s,i}X_{s,j}(1 + K_{p,i}K_{p,j}\delta_{i,j})}{\left(\sum_i X_{s,i}\right)^2} - \left(\frac{\sum_i \sigma_s^o X_{s,i}}{\sum_i X_{s,i}}\right)^2$$

$$
\begin{aligned}
&= (\sigma_s^o)^2 \frac{\sum_i \sum_j X_{s,i} X_{s,j}}{\left(\sum_i X_{s,i}\right)^2} + (\sigma_s^o)^2 \frac{\sum_i \sum_j X_{s,i} X_{s,j} K_{p,i} K_{p,j} \delta_{i,j}}{\left(\sum_i X_{s,i}\right)^2} - (\sigma_s^o)^2 \frac{\sum_i \sum_j X_{s,i} X_{s,j}}{\left(\sum_i X_{s,i}\right)^2} \\
&= (\sigma_s^o)^2 \frac{\sum_i X_{s,i}^2 K_{p,i}^2}{\left(\sum_i X_{s,i}\right)^2} \qquad (23)
\end{aligned}
$$

$$
E[\sigma_e^o] = \frac{\sum_i \sigma_s^o X_{s,i}}{\sum_i X_{s,i}} = \sigma_s^o. \qquad (24)
$$

With the variance and the mean we obtain a compositing equation for the slice K_p.

$$
K_p^2 = \frac{Var[\sigma_e^o]}{(E[\sigma_e^o])^2} = \frac{(\sigma_s^o)^2 \frac{\sum_i X_{s,i}^2 K_{p,i}^2}{\left(\sum_i X_{s,i}\right)^2}}{(\sigma_s^o)^2} = \frac{\sum_i X_{s,i}^2 K_{p,i}^2}{\left(\sum_i X_{s,i}\right)^2}. \qquad (25)
$$

4.3 Compositing K_p equation coefficients

Definitions of the K_p equation coefficients A, B, and C are given in Sections 2 and 3 for egg and slice measurements respectively. Here methods to equate the slice K_p equation coefficients are derived.

4.3.1 A

The A coefficient for the slice measurement differs from the egg A coefficient by the bandwidth used (see Eqs. 3 and 6). For the egg A coefficient the bandwidth used is the 3dB egg bandwidth. This bandwidth is dependent on which beam, inner or outer, and azimuth angle. The slice A coefficient uses the bandwidth of a slice. This value is constant and therefore so is the A coefficient.

Using the compositing method derived for K_p the A^e coefficients can be approximately found from the A^s coefficients.

$$
A^e = \frac{\sum_i X_{s,i}^2 A_i^s}{\left(\sum_i X_{s,i}\right)^2} \qquad (26)
$$

From Figure 1 it can be seen that the composited A^s (all slices approximately equals egg) correctly follows the shape of the A^e coefficient with a small bias. The mean value of this bias is 0.0015.

4.3.2 B and C

The B and C coefficients for the egg and the slice differ by only a constant. This is due to the fact that, for the egg, the bandwidth used to compute these coefficients is B_e, and for the slice the

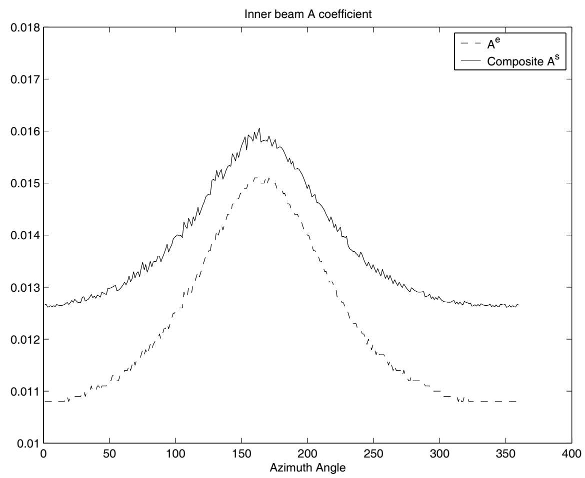


Figure 1: Egg A coefficient and composited slice A coefficient.

bandwidth used is B_s. B_e is the sum of the bandwidths of the slices used in the egg measurement, i.e.

$$B_{egg} = N \cdot B_s. \tag{27}$$

Nominally the center 10 slices are used in the egg calculation, N=10. Using Eqs. 4, 5, 7, and 8 we now have a compositing method for the B and C coefficients.

$$B^e = \frac{2}{B_{egg}T_g} = \frac{2}{N \cdot B_s T_g} = \frac{1}{N}B^s \tag{28}$$

$$C^e = \frac{1}{B_{egg}T_g}\left(1 + \frac{B_{egg}}{B_n}\right) = \frac{1}{N \cdot B_s T_g}\left(1 + \frac{N \cdot B_s}{B_n}\right) \approx \frac{1}{N}C^s \tag{29}$$

where N is the number of slices summed.

4.3.3 Compositing slice SNR

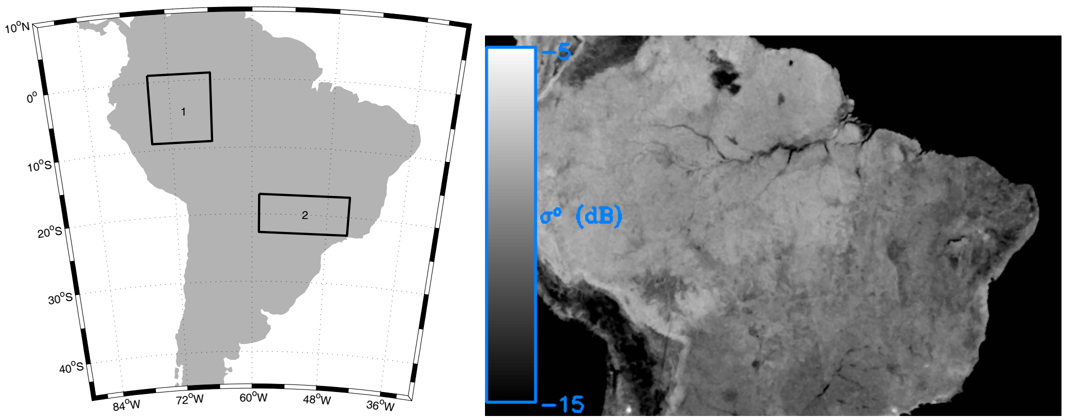
The method for compositing the SNR is the same as the method use to composite the slice σ^o.

$$SNR^e = \frac{\sum_i SNR_i^s X_{s,i}}{\sum_i X_{s,i}} \tag{30}$$

This method introduces a small bias between the egg SNR and the composite slice SNR.

5 Comparison With Empirically Calculated K_p

To verify the methods derived in the previous sections we look at the observed normalized standard deviation of σ^o. This is done by finding regions of semi-homogeneous backscatter, then calculating



(a) (b)

Figure 2: (a) Regions of homogeneous backscatter used in study. (b) SIR images showing backscatter. (quev-a-Ama01-257-260.sir)

the standard deviation of the σ^o measurements and normalizing by the mean (see Eq. 1).

The regions chosen here are shown in Fig. 2. Region 1 is over the Amazon rain forest. This region has a high value of σ^o. Region 2 was chosen for its lower value of σ^o. Note that due to spatial variation in the region the empirical K_p is expected to be larger than the predicted K_p

From Figure 3 it can be seen that Region 1 has a lower empirical K_p than does Region 2. Examination of Fig. 2(b) shows that Region 1 is more homogeneous than is Region 2 therefore explaining the difference in the emprirical K_p. The mean value of predicted K_p reported for the eggs in both regions is approximately the same. This is due to the fact that the SNR is relatively high in both regions and therefore the value of the predicted K_p is mainly dependent on the A coefficient. The difference in the emprical K_p and the predicted K_p comes from the surface inhomogeneity. The empirical K_p is not only a function of the comunication K_p but also a function of the surface K_p.

In Figure 3 the results of the two compositing methods outlined in this report are also shown. Method 1 refers to first calculating the K_p value for every slice using the method outlined in section 3, then compositing each of the K_p values for the slice in a given pulse using Eq. 25. This method shows very good results when compared to the predicted egg K_p.

Method 2 uses the method outlined in Section 4.3. Here each of the coefficients are composited separately. This method is comparable to method 1, with only slightly better results for the added computation.

6 Conclusion

A method for computing K_p for composited slices fro the K_p coefficients in QuikSCAT data is derived and tested.

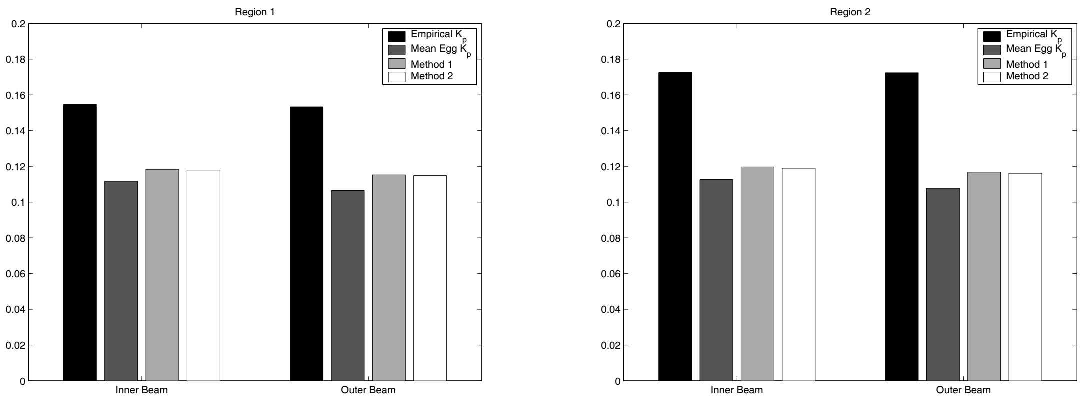


Figure 3: K_p values for both regions in Figure 2. The empirical K_p is found using Eq. 1. The Mean Egg K_p value is the mean value of the egg measurement predicted K_p calculated using the method outlined in section 2. Method 1 refers to the mean value of K_p calculated from the slice K_p using Eq. 25. Method 2 refers to the mean value of K_p calculated from the slice K_p using method outline in section 4.3. The number of measuements for each region and beam are: Region 1 inner beam 25,024, outer beam 23,166, Region 2 inner beam 12,320, and outer beam 12,765

Compositing method for σ^o

$$\sigma_e^o = \frac{\sum\limits_{i=1}^{n} \sigma_{s,i}^o X_{s,i}}{\sum\limits_{i=1}^{n} X_{s,i}}$$

Method 1 for compositing K_p

$$K_p^2 = \frac{\sum\limits_i X_{s,i}^2 K_{p,i}^2}{\left(\sum\limits_i X_{s,i}\right)^2}$$

Method 2 for compositing K_p

$$K_p^2 = A_{comp} + \frac{B_{comp}}{SNR_{comp}} + \frac{C_{comp}}{SNR_{comp}^2}$$

Where

$$A_{comp} = \frac{\sum\limits_i X_{s,i}^2 A_i^s}{\left(\sum\limits_i X_{s,i}\right)^2}$$

$$B_{comp} = \frac{B^s}{N}$$

$$C_{comp} = \frac{C^s}{N}$$

$$SNR_{comp} = \frac{SNR_i^s X_{s,i}}{\sum\limits_i X_{s,i}}$$

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