Range Dependent Phase Gradient Autofocus

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Abstract— The Phase Gradient Autofocus (PGA) algorithm has been widely used in Spotlight Synthetic Aperture Radar (SAR) to remove motion-induced blurs in the images. The PGA algorithm has been proven to be a superior autofocus method. PGA assumes a narrow beam, which is valid for most SAR systems. However, lower altitude SARs have large range dependencies that cannot be ignored. A new phase estimator for PGA is introduced and extended to allow range dependence. An ERS-1 image of Death Valley is used in simulations comparing the new estimator to the widely used maximum likelihood approach and in demonstrating the range-dependent PGA algorithm.

INTRODUCTION AND BACKGROUND

Full focusing of SAR images requires some type of autofocus routine. The Phase Gradient Autofocus (PGA) algorithm has proven to be a superior method for higher order autofocus because it does not assume a model for the phase error. The standard PGA model assumes a small beamwidth in range, which results in a phase error constant in the range direction. Most satellites and other high altitude systems fit this model. However, a low-altitude SAR like YSAR [1] will have range-dependent phase errors. In this paper we extend the algorithm by dropping the narrow beam assumption and introducing range dependencies in the phase error.

There are four main steps in the PGA algorithm. The four steps are center shifting, windowing, phase estimation and iteration. These steps are described in detail in [2, 3, 4]. The Phase Weighted Estimation PGA (PWE-PGA) proposed here differs from earlier algorithms only in the phase estimation step.

A few methods have been proposed for the phase estimation step, with different criteria for optimality. The original PGA algorithm used a linear unbiased minimum variance [2]. The same authors later proposed a method using a maximum likelihood (ML) estimator [3]. This paper proposes a new phase estimation technique which allows extension to a range-dependent algorithm.

RANGE-DEPENDENT PGA

The traditional PGA algorithm described above assumes that the phase error is constant with range and estimates the error in the azimuth direction. A low-altitude SAR system with highly varying incidence angles will exhibit range-dependent effects in the phase errors. This section describes the cause of these range dependencies.

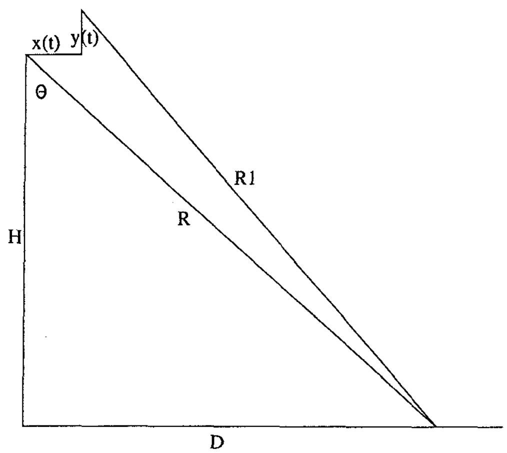

Figure 1: Transverse Motion Geometry

Assume the instrument platform is flying with constant velocity in the direction of increasing z, with the nominal trajectory following $x = y = 0$, as shown in Fig. 1. Then the phase error due to the trajectory errors in the x and y directions can be written as

$$\phi(t, \theta) = \frac{4\pi}{\lambda}(-x(t)\sin(\theta) + y(t)\cos(\theta)), \tag{1}$$

where θ is the incidence angle. The data is stored by range bin instead of incidence angle, so we write the incidence angle for the kth range bin as

$$\theta_k = \cos^{-1}\left(\frac{H}{R_0 + kR}\right). \tag{2}$$

Here H represents the height of the instrument above the topography, R_0 is the range to the zeroth sample,

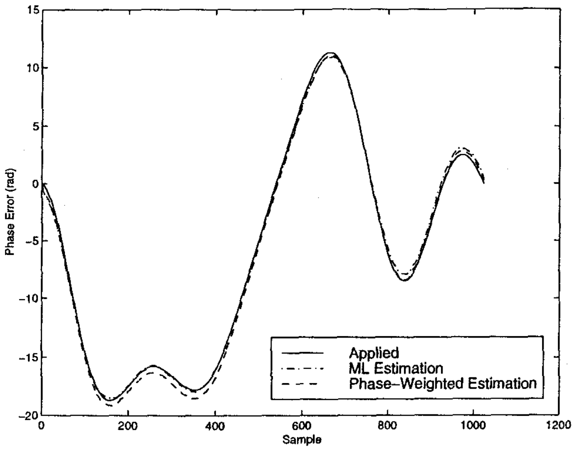

Figure 2: Phase error comparison for non-range-dependent case

and R is the range bin size. Now we have two parameters of phase error to estimate for each azimuth position, $\hat{\phi}_x = -\frac{4\pi}{\lambda}x(t)$ and $\hat{\phi}_y = \frac{4\pi}{\lambda}y(t)$. There is still a large amount of redundancy in the data, so one should be able to effectively estimate these two parameters by adding some kind of range-dependent weighting in the PGA phase estimator. One possibility for the range-dependent weighting is developed in the following section.

PHASE WEIGHTED ESTIMATION

To apply PGA, the gradient of the phase error must be found. The maximum likelihood method is known to be optimal; thus, a first approach would be to apply this method to the range-dependent problem. However, we are not aware of a closed form for the phase estimate. We thus introduce a new algorithm to estimate the phase gradient which allows a simple closed form for a range-dependent version. The phase noise of a sample depends inversely on the magnitude. Thus, our new method weights the phase measurements by the magnitude of the corresponding pixel.

Let g_{kn} denote the image in the range-compressed domain, with k indicating the range bin and n the azimuth bin. Then the estimated phase gradient, denoted $\hat{\phi}_n$, is

$$\hat{\phi}_n = \frac{\sum_{k=0}^{M-1}\left(|(g_{kn}g^*_{k(n-1)})|\angle(g_{kn}g^*_{k(n-1)})\right)}{\sum_{k=0}^{M-1}|(g_{kn}g^*_{k(n-1)})|}. \quad (3)$$

This algorithm can easily by extended to the range-dependent case. The phase weighting remains the same.

We add range weighting and write a vector of equations indexed by range bin k.

$$\hat{\phi}_{xn}\sin(\theta_k) + \hat{\phi}_{yn}\cos(\theta_k) = |g_{kn}g^*_{k(n-1)}|\angle(g_{kn}g^*_{k(n-1)}) \quad (4)$$

This equation is separated into vectors and matrices and written as

$$\mathbf{A}\hat{\phi}_{\mathbf{n}} = \tilde{\phi}_{\mathbf{kn}} \quad (5)$$

where $\mathbf{A}$ is the Mx2 matrix made up of the sine values in the first column and the cosine values in the second, $\hat{\phi}_{\mathbf{n}}$ is the 2x1 vector of phase estimates, and $\tilde{\phi}_{\mathbf{kn}}$ is the Mx1 vector of weighted image phase gradients. This equation is solved using the pseudoinverse of $\mathbf{A}$ to obtain the range-dependent phase gradient estimate. This gradient is then integrated and applied in the same way as in the original PGA algorithm.

RESULTS

The new PWE-PGA algorithm was tested using synthetic phase errors on an ERS-1 image of Death Valley. First a non-range dependent phase error was applied to the image. The resulting blurred image was corrected using the original ML-PGA algorithm and using the new Phase Weighted Estimation PGA. In Fig. 2 the applied phase error is compared with the maximum likelihood and the phase weighted estimation. For this test, the new method is comparable to the ML algorithm but has a slightly larger error.

Figs. 3-5 show a test using range dependent phase errors on the Death Valley image. The original image is shown in Fig. 3(a). The image was then blurred with the range-dependent phase error, resulting in the image in Fig. 3(b). The image was then corrected using the PWE-PGA algorithm, resulting in the restored image in Fig. 3(c). The estimated and applied phase errors for ϕ_x and ϕ_y are shown in Figs. 4 and 5 respectively.

CONCLUSIONS AND FUTURE WORK

The PGA algorithm has been widely used in spotlight SAR to remove motion-induced blur in the images. PGA has been proven to be both a robust, computationally superior autofucus algorithm. The conventional PGA uses a narrow beam approximation to avoid range dependencies. We have introduced a new phase estimator for use in PGA and have extended it to the range-dependent case. Several tests have shown that this algorithm can be successful at removing range-dependent phase errors.

Our planned future work includes a statistical analysis of the new estimator to determine its optimality. We will then further extend the algorithm for application to stripmap mode SAR.

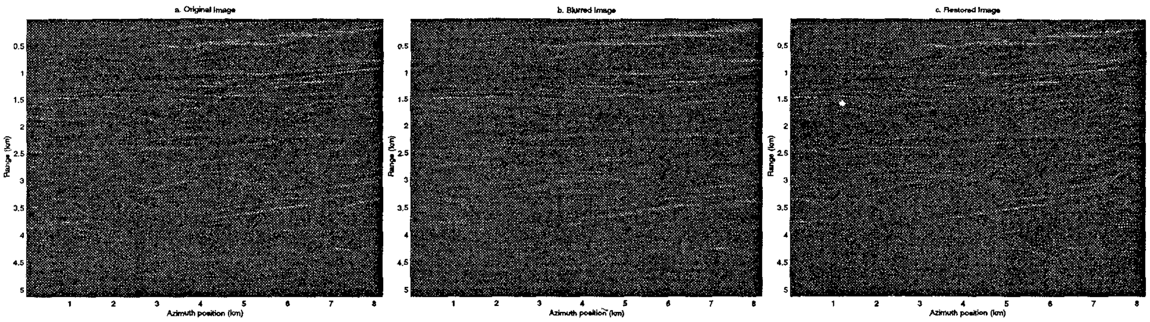

Figure 3: The test image used for range-dependent phase error. (a) The original, focused image of Death Valley from the ERS-1 C-band SAR. (b) The image blurred by a range-dependent phase error. (c) The restored image using the new Phase Weighted Estimation PGA algorithm.

ACKNOWLEDGMENTS

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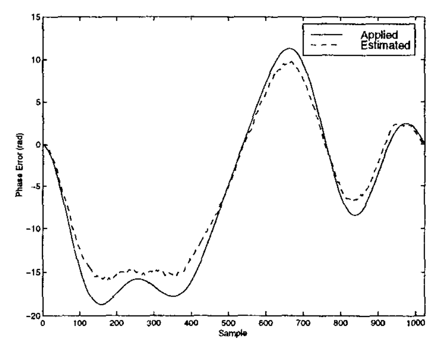

Figure 4: Phase error, x component

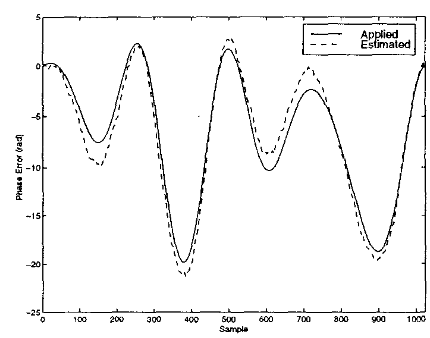

Figure 5: Phase error, y component